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A STUDY OF THE GUIDANCE OF A SPACE VEHICLE RETURNING TO A BRAKING ELLIPSE ABOUT THE EARTH

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SUMMARY

An analysis is made of the guidance of a space vehicle attempting to graze the earth's atmosphere with a specified perigee altitude. Random errors were assumed in the measurement of velocity and flight-path angle and in obtaining the desired thrust impulse. Three methods for scheduling and applying corrective impulses are investigated on the basis of efficiency and accuracy. The first method scheduled thrust corrections as a function of the radial distance from the vehicle to the center of the earth. The second method scheduled the corrections as a function of flight time. The third method scheduled corrections as a function of the angle between perigee and the vehicle position vector. The study showed that the third method provided the best perigee altitude control. However, the economy of the mission indicates that a modification of the third method such that another correction is added near the initial point would provide a more efficient method of scheduling corrective thrust than the present method.

For the three methods studied, an error in the flight-path angle had a predominant effect over errors in velocity and thrust impulse. It was found that, although the trajectory was changed at each point of correction, the radial distance and the instrumentation accuracy at the final correction point before perigee determined the first-pass perigee altitude.

INTRODUCTION

In order to avoid extreme heating and deceleration problems, a space vehicle returning to the earth at super-circular velocities must decrease this velocity before encountering the relatively dense portion of the atmosphere. This velocity decrease could be accomplished by applying reverse thrust but such a procedure is considered impractical because of the large amount of fuel required. A more efficient method would be to cause the vehicle to graze the earth's atmosphere in order to slow the vehicle by aerodynamic braking. This method would probably require several passes to reduce the velocity sufficiently. For this method of capture reference 1 shows the relationship of first-pass perigee altitude to such factors as orbiting time between the first and final pass, heating, and deceleration. Because the number of passes required increases when the first-pass perigee altitude increases, it is desirable for the first-pass perigee altitude to be as low as the limits of heating, deceleration, and accuracy of perigee altitude control allow. A schematic diagram of a braking ellipse trajectory is shown in figure 1.

Although the present study is based on a first-pass perigee altitude tolerance of ±25,000 feet for the nonlifting vehicle described in reference 1, the results are applicable to various reentry corridors, for lifting vehicles.

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In order to obtain a desired first-pass perigee altitude within acceptable limits, the trajectory of the vehicle must be established with great accuracy. For example, a vehicle on a typical approach trajectory (an orbit having an eccentricity close to 1) at a distance of 100,000 miles from the earth and trying to hit a perigee altitude of 250,000 feet would miss this altitude by 25,000 feet (provided no other correction is made) if either a velocity error of about 3 feet per second or a flight-path-angle error of about 0.01° was present. Errors in velocity and flight-path angle of this order of magnitude will probably be masked by the inherent inaccuracies of the instrumentation used to measure these quantities.

One alternative to such extreme accuracy requirements is to provide periodic corrective thrust impulses as the trajectory approaches the earth so that in spite of instrumentation inaccuracies the trajectory can be controlled to the desired perigee altitude. The purpose of this paper is to study the relative performance of three methods of scheduling corrective thrust impulses in the presence of assumed random inaccuracies in measuring velocity and flight-path angle and in obtaining the desired thrust impulse.

SYMBOLS

a	semimajor axis of an ellipse, feet unless otherwise stated
C	correction point
e	eccentricity of an ellipse
g	gravitational constant, 32.2 ft/sec^2

l semilatus rectum of an ellipse, ft

R radius of the earth, 3,960 miles

r radial distance from center of earth to vehicle, ft

r_p radial distance from center of earth to perigee point of flight path, feet unless otherwise stated

t time required to reach perigee, sec

V velocity of space vehicle, ft/sec

V_T magnitude of corrective velocity vector, ft/sec

γ flight-path angle (angle between instantaneous velocity and a line perpendicular to radius from center of earth to space vehicle), deg

θ angle between a line from center of earth to space vehicle and a line from center of earth to perigee point of flight path, deg

α angle between velocity vector and corrective velocity vector, deg

σ_V standard deviation of error in V, ft/sec

σ_γ standard deviation of error in γ, deg

σ_{V_T} standard deviation of error in V_T, percent

Δt change in t, sec

$\Delta\gamma$ change in γ, deg

$\Delta\theta$ change in θ, deg

Δr change in r, ft

ϵ_V magnitude of error in V, ft/sec

ϵ_γ magnitude of error in γ, deg

ϵ_{V_T} magnitude of error in V_T, ft/sec

h_p perigee altitude, ft

Subscripts:

o conditions that define initial trajectory

1,2,3,...10 order of corrections where 1 is correction at initial position

R result of applying a correction

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METHOD OF ANALYSIS

Approach Conditions and Assumptions

This study is concerned with the portion of the trajectory of a space vehicle approaching the earth beginning at a distance of about 100,000 miles from the center of the earth and ending at the first-pass perigee altitude. In all cases considered the space vehicle is approaching the earth on an elliptical path with an eccentricity of almost 1 and traveling at the appropriate elliptic velocity.

Time, temperature, and deceleration are limiting factors for a braking ellipse capture. Reference 1 shows that orbiting time between the first and final passes increases drastically as the first-pass perigee altitude increases and the temperature and deceleration increase as the first-pass perigee altitude decreases. For the vehicle described in reference 1 having a perigee velocity of about 36,000 feet per second, orbiting time increased from 2 to 11 days when the perigee altitude increased from 250,000 to 275,000 feet and equilibrium wall temperature increased from 2,250° F to 2,600° F when the perigee altitude decreased from 250,000 to 225,000 feet. For this study based on this information, 250,000 feet was chosen for the desired first-pass perigee altitude and control of the first-pass perigee altitude within ±25,000 feet was assumed to be acceptable.

The following assumptions are made in this study:

(1) The earth is spherical.

(2) Motion is considered only in the plane of the orbit for a nonrotating earth.

(3) The space vehicle is close enough to the earth so that the gravitation fields of all other bodies may be neglected.

The basic technique of this study is to apply corrections at given intervals along the flight path in order to provide the space vehicle with an acceptable perigee altitude control. At each correction point the orbital characteristics, from the measured values of V and γ (obtained by adding an assumed error to the true value), are calculated. Calculations are then made to determine the optimum direction and magnitude of corrective velocity required to correct the perigee altitude to the desired first-pass perigee altitude. After adding an assumed error in V_T, the thrust impulse is applied in the optimum direction.

The technique used in selecting errors to represent instrumentation inaccuracies in measuring the desired variables was as follows. The range between a positive and a negative assumed error was divided into 100 equal increments. These increments were distributed normally along a table of numbers from 1 to 1×10^6 such that every number between 1 and 1×10^6 had a specific magnitude of error assigned to it. A random number process was used to select a number $\left(\text{between 1 and } 1 \times 10^6\right)$ and the error assigned to this number in the table was used to represent the inaccuracy of measuring the desired variable. This technique, generally referred to as the Monte Carlo technique, is described in more detail in reference 2.

Equations

In accordance with the foregoing assumptions, the flight path of a space vehicle approaching the earth on an elliptical path is described by the following relations. (See ref. 3.)

$$r = \frac{l}{1 + e \cos \theta} \tag{1}$$

$$V = \sqrt{gR^2\left(\frac{2}{r} - \frac{1}{a}\right)} \tag{2}$$

$$\cos \gamma = \sqrt{\frac{l g R^2}{r^2 V^2}} \tag{3}$$

$$e = \sqrt{1 - \frac{l}{a}} \tag{4}$$

$$r_p = a(1 - e) \tag{5}$$

The present study is based on the application of a thrust impulse in the optimum direction at a given radial distance in order to correct the perigee altitude. When a thrust impulse is applied, the velocity and flight-path angle (dependent upon a known value of r) of the vehicle are changed. This change in V and γ defines a new trajectory with a different perigee attitude. Therefore, the following analysis is made in order to determine (at any radial distance) the direction to apply corrective thrust that will require the minimum thrust impulse to produce a desired change in perigee altitude.

If equations (2), (3), and (4) are rearranged and then substituted into equation (5) an expression for the perigee distance r_p (measured from the center of the earth) in terms of the trajectory variables r, V, and γ is obtained.

$$r_p = \frac{gR^2\left[1 - \sqrt{1 - \frac{r^2V^2\cos^2\gamma\left(2g\frac{R^2}{r} - V^2\right)}{\left(gR^2\right)^2}}\right]}{2g\frac{R^2}{r} - V^2} \tag{6}$$

The total derivative of $r_p(V,\gamma)$ is derived in the appendix and the result is given by the following relation:

$$dr_p(V,\gamma) = \frac{\partial r_p}{\partial \gamma}\,d\gamma + \frac{\partial r_p}{\partial V}\,dV \tag{7}$$

where

$$\frac{\partial r_p}{\partial \gamma} = -\frac{l}{e}\tan\gamma$$

$$\frac{\partial r_p}{\partial V} = \frac{2gR^2(1 - e)}{e\left(2g\frac{R^2}{r} - V^2\right)^2}\left[\frac{g\frac{R^2}{r}(1 + e)}{V} - V\right]$$

The following relations for dV and $d\gamma$ are obtained from a diagram of the velocity vector and the corrective velocity vector shown in figure 2. These relations are true when ϵ_γ is small and ϵ_V is small in comparison with V.

$$dV = V_T\cos\alpha \tag{8}$$

$$d\gamma = \frac{V_T \sin \alpha}{V} \tag{9}$$

Substituting these expressions for dV and $d\gamma$ into equation (7) gives the following expression:

$$dr_p = \frac{\partial r_p}{\partial \gamma} \frac{V_T \sin \alpha}{V} + \frac{\partial r_p}{\partial V} V_T \cos \alpha \tag{10}$$

The maximum change in r_p is obtained when the corrective velocity is made in the direction defined by

$$\tan \alpha = \frac{\dfrac{\partial r_p}{\partial \gamma}}{\dfrac{\partial r_p}{\partial V} V} \tag{11}$$

From equations (7) to (11), a minimum value of the magnitude of the corrective velocity vector may be found to produce a given change in the perigee altitude.

Errors

Little data are available on the accuracy of measuring the velocity and flight-path angle of an object at a great distance from the earth. The assumed errors of this report are based on available information and it is believed that they are of the correct order of magnitude.

The errors in measuring velocity and flight-path angle and the error in applying corrective thrust are assumed to have a normal distribution. In order to determine the effects of an error in measuring V, the standard deviation of the error in V was varied from 1 to 10 feet per second while the errors in γ and V_T were assumed to be zero and then held at $\sigma_\gamma = 0.0125^\circ$ and $\sigma_{V_T} = 1.3$ percent. In order to determine the effects of an error in measuring γ, the standard deviation of the error in γ was varied from 0.00625° to 0.0375° while the errors in V and V_T were first assumed to be zero and then were $\sigma_V = 1$ ft/sec and $\sigma_{V_T} = 1.3$ percent. In order to determine the effects of error in the magnitude of the corrective velocity, the standard deviation of the error in V_T was varied from 1.3 to 10.4 percent while the errors in V and γ were $\sigma_V = 1$ ft/sec and $\sigma_\gamma = 0.0125^\circ$. In this investigation the errors in V and γ were first assumed to be dependent upon range and then invariant with range.

In the solution of equations (6) to (11), the magnitude of the errors at each correction point was determined by the application of the Monte Carlo technique (ref. 2) whereby a random number process based upon the error distribution is used to select the error magnitude.

Initial conditions were assumed such that without corrections perigee radii of (1) $R + 250{,}000$ feet where $R = 3{,}960$ miles, (2) 6,000 miles, and (3) 8,000 miles would be obtained. Studies were made of runs which started with the initial conditions where $r_{p_o} = R + 250{,}000$ feet in order to determine the effects of instrumentation inaccuracies. Runs of the initial conditions where $r_{p_o} = 6{,}000$ miles and $r_{p_o} = 8{,}000$ miles were studied in order to determine the effects of instrumentation inaccuracies when relatively large corrections would be required to obtain a perigee altitude of 250,000 feet.

In order to obtain the distribution of the perigee altitude and total corrective thrust, 1,100 runs were made for each case.

METHODS OF CONTROL

Three methods of scheduling corrective thrust before the space vehicle made its first pass through the atmosphere were investigated. Schematic diagrams of the three trajectories, defined by $a_o = 100{,}000$ miles and the three values of r_{p_o} of this study and presented so that the perigee altitude of the three would be above the same point on the surface of the earth, are shown in figure 3. Also shown in figure 3 are the initial points for these trajectories and the correction points along the trajectory where $r_{p_o} = R + 250{,}000$ feet for the three methods of scheduling corrective thrust.

The first method of scheduling corrective thrust, which is referred to as the radial distance method, was to apply a correction whenever the distance from the center of the earth to the space vehicle decreased by a certain amount. The initial correction point for this method was at a radius of 100,000 miles. For the radial distance method, radial increments of 5,000, 7,500, 10,000, 12,500, and 15,000 miles were studied but results for only the 10,000-mile increments are presented. The 10,000-mile increments were chosen because (1) the total V_T increased rapidly as the increments decreased below 10,000 miles, and (2) the perigee altitude band increased in width with little or no change in total V_T as the increments increased above 10,000 miles.

The second method of scheduling corrective thrust was to apply a correction at constant time intervals. It was found that, when small constant time intervals along the flight path were used in order to have several corrections relatively close to perigee, most of the corrections were applied in a region where few corrections were needed. Therefore this method was modified in order to reduce the number of corrections by dividing the flight path into time segments and using different time intervals in each segment. The two divisions of the flight path studied, referred to as time method schedule A and time method schedule B, are given in the following tables:

Time method schedule A	
Correction	Time required to reach perigee, hours
C_1	15
C_2	10
C_3	7.5
C_4	5
C_5	4
C_6	3
C_7	2
C_8	1.5
C_9	1
C_{10}	0.5

Time method schedule B	
Correction	Time required to reach perigee, hours
C_1	17
C_2	16
C_3	11
C_4	6
C_5	3.5
C_6	1
C_7	0.666
C_8	0.333

The third method of scheduling corrective thrust, which is referred to as the angular method, was to apply a correction whenever the angle between the radius to the space vehicle and the perigee radius decreased a given amount. The initial correction point for this method was at $\theta = 160^\circ$. Angular increments of 5°, 10°, 15°, 20°, 30°, and 45° were studied but increments of less than 30° led to excessively large total thrust requirement without improving the accuracy; therefore only increments of 30° and 45° were selected for this method of scheduling corrective thrust.

RESULTS AND DISCUSSION

General Discussion

The results of this study, presented in figures 4 to 25, are shown as probability curves. The perigee altitude probability curves where each curve is based upon 1,100 runs are presented such that the probability of the perigee altitude being greater than a given value of r_p can be read directly from the figure. The total corrective velocity probability curves where each curve is based upon 1,100 runs are presented such that the probability of the total corrective velocity being less than a given total value of V_T can be read directly from the figure.

Although all the results of the present study are presented for a desired first-pass perigee altitude of 250,000 feet, these results are applicable to any desired perigee altitude because the equations of motion and the equation for the change in perigee distance (eq. 10) are not dependent upon a specific value of r_p. For example, if 300,000 feet had been chosen as the desired first-pass perigee altitude, the perigee altitude scale of figure 4(a) would be from 250,000 to 350,000 feet with 300,000 feet in the center.

As noted previously, a perigee altitude band 50,000 feet wide shall be considered the maximum acceptable perigee altitude band for a space vehicle. It should be noted, however, that an acceptable perigee altitude band will depend upon the characteristics of the space vehicle. (See ref. 1.)

In order to show an example of the magnitudes of error in V, γ, and V_T selected by the random process, typical runs for the three methods of scheduling corrective thrust were selected. These runs were from cases where $a_o = 100{,}000$ miles, $r_{p_o} = 6{,}000$ miles, $\sigma_V = 1r/10{,}000$ ft/sec, $\sigma_\gamma = 0.0125r/10{,}000$ degree, and $\sigma_{V_T} = 1.3$ percent. Table I lists the errors and other information at each correction point for the radial distance method, time method schedule B, and angular method ($\Delta\theta = 30^\circ$). The perigee altitude h_p listed in the table is the perigee altitude that the vehicle would obtain if no other corrections were made. It should be noted that the velocity at perigee for all three methods of scheduling corrective thrust would be approximately the same.

Effect of Semimajor Axis

Cases were analyzed for three values of a_o in order to determine the effect of the semimajor axis on the distribution of perigee altitude. The values of a_o used were 100,000 miles, 150,000 miles, and 200,000 miles. These values of a_o were used in the three sets of initial conditions for the three methods of scheduling corrective thrust. For the initial trajectories where $r_{p_o} = R + 250{,}000$ feet and where $a_o = 100{,}000$ miles, 150,000 miles, and 200,000 miles, the velocities at $r = 100{,}000$ miles are 5,160 ft/sec, 5,828 ft/sec, and 6,410 ft/sec, respectively, for the three trajectories. As a_o increases, the velocity approaches escape velocity, 7,295 ft/sec at $r = 100{,}000$ miles. The results of cases where the errors in V, γ, and V_T were represented by $\sigma_V = 1r/10{,}000$ ft/sec, $\sigma_\gamma = 0.0125r/10{,}000$ degree, and $\sigma_{V_T} = 1.3$ percent are shown in figure 4. The results of cases where the errors are represented by $\sigma_V = 1$ ft/sec, $\sigma_\gamma = 0.0125^\circ$, and $\sigma_{V_T} = 1.3$ percent are shown in figure 5. These results show that changing a_o had a negligible effect on the perigee altitude. Although not shown, the total corrective velocity requirements were approximately the same for the three values of a_o studied. Thus the remainder of this study was made with $a_o = 100{,}000$ miles.

Figures 4 and 5 also show that, for a given method of scheduling corrective thrust, the same perigee altitude band is indicated for cases using the three sets of initial conditions. This was found to be true for all combinations of errors studied; therefore, the perigee altitude probability curves for cases where the initial conditions were such that $r_{p_o} = 6{,}000$ and 8,000 miles will not be included in the remainder of the report.

Effects of an Error in Measuring Velocity

Errors assumed to be dependent upon range.- A series of cases where the error in V was varied were analyzed in order to determine the effect on the perigee altitude and the total corrective thrust. This series included cases where the errors in γ and V_T were first assumed to be zero and then $\sigma_\gamma = 0.0125r/10{,}000$ degree and $\sigma_{V_T} = 1.3$ percent while the error in V varied. Figures 6 and 7 show the results of cases where no errors in γ and V_T were assumed and the errors in V were

$\sigma_V = 1r/10{,}000$ ft/sec, $2r/10{,}000$ ft/sec, and $3r/10{,}000$ ft/sec. The results of cases where the radial distance method and the time method schedule B were used show increases of about 8,000 and 3,000 feet, respectively, in the width of the perigee altitude band for an increase of 1 ft/sec in σ_V. When the angular method was used, the effect upon r_p of increasing σ_V was too small to be detected in figure 6.

Figure 7 shows that for all three methods of scheduling corrective thrust approximately 40 feet per second of total corrective velocity may be required for each increase of 1 foot per second in σ_V. This increment was taken at 100-percent probability.

The results of cases where the errors in γ and V_T were $\sigma_\gamma = 0.0125r/10{,}000$ degree and $\sigma_{V_T} = 1.3$ percent and the errors in V were $\sigma_V = 1r/10{,}000$ ft/sec, $2r/10{,}000$ ft/sec, $3r/10{,}000$ ft/sec, and $10r/10{,}000$ ft/sec are given in figures 8 and 9. Figure 8 indicates that, in the presence of errors in γ and V_T, an error in V that is dependent upon range has a negligible effect on the perigee altitude. Figure 9 shows that in the presence of errors in γ and V_T an error in V of $\sigma_V = 3r/10{,}000$ ft/sec or less has little effect on the economy of the mission but that an error of $\sigma_V = 10r/10{,}000$ ft/sec has a serious effect on the economy.

Errors assumed invariant with range.- The results of cases where no errors in γ and V_T were assumed and the errors in V were $\sigma_V = 1$ ft/sec, 2 ft/sec, and 3 ft/sec are shown in figures 10 and 11. For an increase of 1 foot per second in σ_V, the width of the perigee altitude band increased about 8,000 and 3,000 feet when the radial distance method and the time method schedule B, respectively, were used. These results (fig. 11) indicate that a total V_T of less than 10 ft/sec will be required for each error of 1 ft/sec in σ_V (not dependent upon range).

Figures 12 and 13 give the results of cases where the errors in γ and V_T were $\sigma_\gamma = 0.0125^\circ$ and $\sigma_{V_T} = 1.3$ percent and the errors in V were $\sigma_V = 1$ ft/sec, 2 ft/sec, 3 ft/sec, and 10 ft/sec. In the presence of errors in γ and V_T, an error in V of $\sigma_V = 3$ ft/sec or less had a very small effect on both the perigee altitude and the total corrective velocity. However, figure 13 shows that, when the error in V is increased from $\sigma_V = 3$ ft/sec to $\sigma_V = 10$ ft/sec, the total V_T requirements show increases on the order of 40 feet per second.

Effects of an Error in Measuring γ

Errors assumed to be dependent upon range.- A series of cases where the error in γ was varied were analyzed in order to determine the effects on the perigee altitude and total corrective velocity. Figures 14 and 15 give the results of cases where no errors in V and V_T were assumed and the errors in γ were $\sigma_\gamma = 0.00625r/10{,}000$ degree, $0.0125r/10{,}000$ degree, $0.0250r/10{,}000$ degree, and $0.0375r/10{,}000$ degree. For each increase in σ_γ (dependent upon range) of 0.0125°, the width of the perigee altitude band increased about 60,000 feet when the radial distance method was used, 35,000 feet when the time method schedule B was used, and 2,500 feet when the angular method was used. The total V_T requirements of all three methods of scheduling corrective thrust increased about 200 feet per second each time σ_γ (dependent upon range) increased 0.0125°.

Figures 16 and 17 give the results of cases where the errors in V and V_T were $\sigma_V = 1r/10{,}000$ ft/sec and $\sigma_{V_T} = 1.3$ percent and the errors in γ were $\sigma_\gamma = 0.00625r/10{,}000$ degree, $0.0125r/10{,}000$ degree, $0.0250r/10{,}000$ degree, and $0.0375r/10{,}000$ degree. A comparison of figures 14 and 15 with figures 16 and 17 indicates that, when an error in γ of $\sigma_\gamma = 0.00625r/10{,}000$ degree is present, the effect on r_p and total V_T of adding errors in V and V_T of $\sigma_V = 1r/10{,}000$ ft/sec and $\sigma_{V_T} = 1.3$ percent is negligible.

Errors assumed invariant with range.- The results of cases where no errors were assumed in V and V_T and errors in γ were $\sigma_\gamma = 0.00625^\circ$, 0.0125°, 0.0250°, and 0.0375° are shown in figures 18 and 19. For each increase in σ_γ (not dependent upon range) of 0.0125° the perigee altitude band increased in width about 60,000 feet when the radial distance method was used, about 45,000 feet when the time method schedule B was used, and about 6,000 feet when the angular method was used. The total V_T that may be required to accomplish the mission increased about 50 ft/sec when either the radial distance method or the time method schedule B was used and about 120 ft/sec when the angular method was used each time σ_γ (not dependent upon range) increased 0.0125°.

Figures 20 and 21 give the results of cases where errors in V and V_T were $\sigma_V = 1$ ft/sec and $\sigma_{V_T} = 1.3$ percent and the errors in γ were $\sigma_\gamma = 0.00625^\circ$, 0.0125°, 0.0250°, and 0.0375°. These results show the same distribution of r_p and total V_T as that shown in figures 18 and 19.

Effects of an Error in the Magnitude of V_T

A series of cases where the error in V_T was varied were analyzed in order to determine the effects of an error in the magnitude of V_T. The assumed errors for V_T were $\sigma_{V_T} = 1.3$, 2.6, 3.9, and 10.4 percent. Figures 22 and 23 give the results of cases where the errors in V and γ were $\sigma_V = 1r/10{,}000$ ft/sec and $\sigma_\gamma = 0.0125r/10{,}000$ degree while the error in V_T varied. Figures 24 and 25 give the results of cases where the errors in V and γ were $\sigma_V = 1$ ft/sec and $\sigma_\gamma = 0.0125^\circ$ while the error in V_T varied. These results indicate that an error in the magnitude of V_T has a very small effect on the perigee altitude. (See figs. 22 and 24.) The total corrective velocity distributions (figs. 23 and 25) show that a 10.4-percent error in V_T has a very small effect on the economy of the mission when either the radial distance method or the time method schedule B is used. However, when the angular method is used for cases where the initial conditions were such that $r_{p_o} = 6{,}000$ and 8,000 miles, large increases in the total V_T are observed when the error in V_T increased from 3.9 percent to 10.4 percent.

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Overall Comparison of Three Methods

<u>Perigee altitude control</u>.- The perigee altitude probability curves in figure 4, results of cases where the errors were represented by $\sigma_V = 1r/10{,}000$ ft/sec, $\sigma_\gamma = 0.0125r/10{,}000$ degree, and $\sigma_{V_T} = 1.3$ percent, are used to compare the perigee altitude control of the three methods of scheduling corrective thrust. The radial distance method (fig. 4(a)) gave a poor perigee altitude control that has about 95-percent probability of obtaining a perigee altitude within ±25,000 feet of the desired perigee altitude. The time method schedule B (fig. 4(c)) gave a poor but acceptable perigee altitude control. The angular method using $\Delta\theta = 30^\circ$ or 45° (figs. 4(d) and 4(e)) gave excellent perigee altitude control. This comparison shows that the angular method, from a perigee-altitude control viewpoint, is superior to the other method studied. The superior perigee-altitude control of the angular method may be seen by comparing the perigee-altitude probability curves in other figures.

A comparison of results for the three methods of scheduling corrective thrust indicated that the angular method gave the best perigee-altitude control because the radial distance to the final correction point for this method was less than that for the other methods studied.

In order to investigate this point, additional cases for the three methods of scheduling corrective thrust were analyzed where the radial distance to the final correction point before perigee was approximately the same for all three methods. For convenience, the radial distance to the last correction point for the time method schedule B ($r \approx 6{,}725$ miles) was selected for these cases. For the radial distance method, r_o was changed to 96,725 miles so that the radial distance at the final correction point would be 6,725 miles when $\Delta r = 10{,}000$ miles. For the angular method (where $\Delta\theta = 30^{\circ}$), θ_o was changed to 170° and the final correction before perigee was made at $\theta = 80^{\circ}$ (where $r \approx 6{,}730$ miles). The results of these cases (fig. 26) show approximately the same perigee-altitude band for the three methods of scheduling corrective thrust. It is therefore concluded that the predominant reason for the superior perigee-altitude control of the angular method is that the radial distance to the last correction point for this method is less than that for the other two methods studied. In general, it may be stated that the perigee altitude is primarily determined by the radial distance and instrumentation inaccuracies at the final correction point. The ordered sequence of corrections prior to the final correction is secondary in accuracy but directly affects the total corrective velocity required. Corrections based on angular increments where the number of corrections increase with proximity to the earth appears to be a desirable scheme during the terminal phase.

Corrective velocity requirements.- A comparison of the probability curves of the total corrective velocity for the condition where $r_{p_o} = R + 250{,}000$ feet and for given accuracies indicate that all three methods of scheduling corrective thrust have approximately the same corrective thrust requirements. By comparing the corrective velocity probability curves for the initial conditions where $r_{p_o} = 6{,}000$ miles and 8,000 miles at given accuracies, it is seen that each dr_{p_o} of 1,000 miles require a V_T of about 150 feet per second when either the radial distance method or the time method schedule B is used and about 225 feet per second when the angular method ($\Delta\theta = 30^{\circ}$) is used.

A larger V_T is required to correct an initial error in r_p when the angular method is used because of the small-angle approximation used in calculating V_T. Because of the relatively large changes in V and γ required to correct an error of 1,000 miles (or larger) in r_{p_o}, the relations for $d\gamma$ and dV (eqs. (8) and (9)) introduce errors in calculating V_T that cause the value of V_T calculated to be smaller than the value of V_T needed to correct the error in r_{p_o}. Thus the first thrust impulse does not entirely correct the initial error in r_{p_o}

and a second impulse is required before the space vehicle is approximately on the desired flight path. When either the radial distance method or the time method schedule B is used, the second correction point is relatively close to the initial point and the value of V_T required to make the second correction in r_p is small compared with the first correction. However, when the angular method is used, the second correction point is at a great distance from the initial point and the value of V_T required to make the second correction in r_p is large. This effect of the location of the second correction point on V_T can be seen by comparing the values of r, h_p, and V_T from runs for the three methods of scheduling corrective thrust given in table I.

The difference between the total corrective velocity requirements for the angular method and the other two methods is about the same as the difference between the corrective velocity used to make the second correction for the respective methods. Thus, when there is an error in r_{p_o} of 1,000 miles or more, a modification of the angular method such that an additional correction is applied close to the initial point would lower the total corrective velocity requirements for this method.

A comparison of results of cases where the errors in V and γ that are dependent upon range and the errors in V and γ that are not dependent upon range indicate that the corrective thrust requirements are lower for the cases where the errors are not dependent upon range.

CONCLUSIONS

Results of a study of the guidance of a space vehicle returning to a braking ellipse about the earth where three methods of scheduling corrective thrust were compared in terms of errors in perigee altitude and total corrective velocity required led to the following conclusions:

1. The effect on perigee altitude and total corrective velocity of an error in the flight-path angle is predominant over the effect of errors in the magnitude of both the space vehicle's velocity and the corrective velocity.

2. Although the trajectory is changed at each correction point along the flight path, the radial distance and the instrumentation inaccuracies at the final correction point before reaching perigee determined the first-pass perigee altitude. Therefore, it is desirable to make the final correction relatively close to perigee.

3. The effect on perigee altitude and total corrective thrust of varying the semimajor axis of the initial trajectory from 100,000 to 200,000 miles was negligible.

4. The accuracy with which the desired perigee altitude can be obtained is not dependent on the value of the original (that is, if no corrections are made to the original trajectory) perigee altitude.

5. The angular method of scheduling corrective thrust (applying a correction each time the angle between the radius to the space vehicle and the perigee radius decreased 30°) provided excellent perigee-altitude control for all errors studied.

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6. When the difference between the desired first-pass perigee altitude and the initial perigee altitude is 1,000 miles (or more), a modification of the angular method such that an additional correction is applied close to the initial point would lower the total corrective velocity requirements for this method of scheduling corrective thrust.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., September 16, 1959.

APPENDIX

DEVIATIONS OF PERIGEE ALTITUDE CAUSED BY CHANGES IN γ and V

The equation for the perigee distance of an elliptical orbit in terms of r, V, and γ is

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$$r_p = \frac{gR^2 - gR^2\sqrt{1 - \frac{r^2V^2\cos^2\gamma\left(2g\frac{R^2}{r} - V^2\right)}{\left(gR^2\right)^2}}}{2g\frac{R^2}{r} - V^2} \tag{A1}$$

Since the present study is concerned with the change in r_p caused by changes in V and γ at a given radial distance, the variable r in equation (A1) is considered a known quantity.

The following expression may be obtained:

$$dr_p = \frac{\partial r_p}{\partial \gamma} d\gamma + \frac{\partial r_p}{\partial V} dV \tag{A2}$$

The partial derivative of r_p with respect to γ is obtained by differentiating equation (A1)

$$\frac{\partial r_p}{\partial \gamma} = -\frac{gR^2}{2g\frac{R^2}{r} - V} \frac{\frac{4r^2V^2g\frac{R^2}{r}}{\left(gR^2\right)^2}\sin\gamma\cos\gamma - \frac{2r^2V^4}{\left(gR^2\right)^2}\sin\gamma\cos\gamma}{2\sqrt{1 - \frac{r^2V^2\cos^2\gamma\left(2g\frac{R^2}{r} - V^2\right)}{\left(gR^2\right)^2}}}$$

By multiplying the numerator by $\frac{\cos\gamma}{\cos\gamma}$ and substituting l for $\frac{r^2V^2\cos^2\gamma}{gR^2}$ (eq. (3)), this expression becomes

$$\frac{\partial r_p}{\partial \gamma} = - \frac{l\frac{\sin \gamma}{\cos \gamma}}{\sqrt{1 - \frac{r^2V^2\cos^2\gamma\left(2g\frac{R^2}{r} - V^2\right)}{\left(gR^2\right)^2}}}$$

If

$$e = \sqrt{1 - \frac{r^2V^2\cos^2\gamma\left(2g\frac{R^2}{r} - V^2\right)}{\left(gR^2\right)^2}}$$

then

$$\frac{\partial r_p}{\partial \gamma} = - \frac{l}{e} \tan \gamma$$

The partial derivative of r_p with respect to V is obtained by differentiating equation (A1) as

$$\frac{\partial r_p}{\partial V} = \frac{2g\frac{R^2}{r} - V^2}{\left(2g\frac{R^2}{r} - V^2\right)^2} \frac{2\left(Vr^2\cos^2\gamma g\frac{R^2}{r} - V^3r^2\cos^2\gamma\right)}{gR^2\sqrt{1 - \frac{r^2V^2\cos^2\gamma\left(2g\frac{R^2}{r} - V^2\right)}{\left(gR^2\right)^2}}}$$

$$- \frac{gR^2 - gR^2\sqrt{1 - \frac{r^2V^2\cos^2\gamma\left(2g\frac{R^2}{r} - V^2\right)}{\left(gR^2\right)^2}}}{\left(2g\frac{R^2}{r} - V^2\right)^2}(-2V)$$

Substituting e and $1 - e^2 = \dfrac{V^2r^2\cos^2\gamma\left(2g\frac{R^2}{r} - V^2\right)}{\left(gR^2\right)^2}$, and simplifying yield

$$\frac{\partial r_p}{\partial V} = \frac{2gR^2(1 - e)}{e\left(2g\frac{R^2}{r} - V^2\right)^2}\left[\frac{\left(1 - e^2\right)\left(g\frac{R^2}{r} - V^2\right)}{V(1 - e)} + Ve\right]$$

$$= \frac{2gR^2(1 - e)}{e\left(2g\frac{R^2}{r} - V^2\right)^2}\left[\frac{g\frac{R^2}{r}(1 + e)}{V} - V\right]$$

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2. Blumstein, Alfred: Application of Monte Carlo Techniques to the Analysis of the Ground Controlled Approach System. Rep. No. JA-848-G-2, Cornell Aero. Lab., Inc., Feb. 1, 1954.

3. Moulton, Forest Ray: An Introduction to Celestial Mechanics. Second rev. ed., The Macmillan Co., c.1914.

TABLE I.- TYPICAL RUNS FROM CASES WHERE a_o = 100,000 MILES, r_{p_o} = 6,000 MILES, σ_V = 1r/10,000 FT/SEC, σ_γ = 0.0125r/10,000 DEGREE, AND σ_{V_T} = 1.3 PERCENT

C	r, miles	ϵ_V, ft/sec	ϵ_γ, deg	ϵ_{V_T}, ft/sec	h_p, ft	V_R, ft/sec	γ_R, deg	V_T, ft/sec (a)	α, deg	a, miles	e
Radial distance method											
1	100,000	-10.80	-0.0550	-2.95	1,176,602	5,073.18	-73.0820	284.02	250.6383	96,762	0.9568
2	90,000	6.12	.0225	-.45	260,005	5,616.71	-73.3789	33.73	253.4408	96,387	.9584
3	80,000	3.52	.0280	-.01	296,023	6,240.95	-73.1510	1.51	73.6450	96,406	.9583
4	70,000	-1.40	.0735	.02	436,664	6,963.21	-72.6707	6.71	73.4137	96,499	.9581
5	60,000	-.24	-.0690	-.11	88,021	7,815.11	-72.1136	19.51	252.7159	96,185	.9587
6	50,000	-.20	.1325	-.32	534,231	8,887.70	-70.8859	29.98	72.2614	96,755	.9580
7	40,000	-.48	-.0340	-.55	177,824	10,267.31	-69.4274	30.00	250.8195	96,051	.9584
8	30,000	3.72	.0045	.04	246,665	12,241.96	-66.8221	7.81	69.2244	96,286	.9584
9	20,000	2.16	.0070	.03	255,207	21,600.00	-62.0989	1.49	66.5883	96,350	.9584
10	10,000	-1.00	-.0145	-.12	237,854	22,467.46	-49.9687	6.74	242.7842	95,869	.9582
Time method schedule B											
1	96,880	7.36	-0.0436	-2.51	1,024,488	5,232.10	-73.1266	298.34	250.5569	96,491	0.9569
2	94,633	6.44	-.0615	.05	48,265	5,345.67	-73.4759	34.31	253.3904	96,130	.9587
3	75,496	.30	-.1019	-.02	-3,524	6,544.66	-73.0914	2.31	253.4925	96,100	.9588
4	51,130	-.20	-.0741	.06	89,272	8,745.44	-71.2111	6.13	72.1988	96,214	.9587
5	35,473	2.70	.0479	.73	339,343	11,074.10	-68.3596	23.89	70.3348	96,838	.9584
6	14,372	.86	.0194	-.01	271,238	18,514.31	-57.0770	17.02	244.5016	95,893	.9582
7	10,684	1.28	.0069	-.03	256,315	21,693.15	-51.4088	5.32	243.0148	95,533	.9580
8	6,722	.40	.0091	.00	255,278	27,646.46	-38.9358	.73	242.9762	95,470	.9580
Angular method ($\Delta\theta$ = 30°)											
1	99,752	-5.19	0.0249	-8.15	1,188,319	5,086.07	-73.0786	284.34	250.6827	96,758	0.9567
2	21,270	.43	.0330	2.61	325,782	14,874.43	-62.7763	138.57	246.6920	91,386	.9560
3	9,432	.79	.0127	-.56	258,730	23,127.55	-48.5258	28.08	242.5479	89,548	.9552
4	5,908	.64	.0050	-.03	252,230	29,521.17	-34.0917	5.88	244.1163	89,093	.9550
5	4,525	-.63	.0111	-.02	253,057	33,868.96	-19.5174	1.73	71.5940	89,203	.9551
6	4,038	-.57	.0063	.17	250,414	35,900.83	-4.9252	25.42	264.0214	88,641	.9548

[a]Total V_T for radial distance method is 421.51 ft/sec; for time method schedule B, 388.05 ft/sec; and for angular method ($\Delta\theta$ = 30°), 484.02 ft/sec.

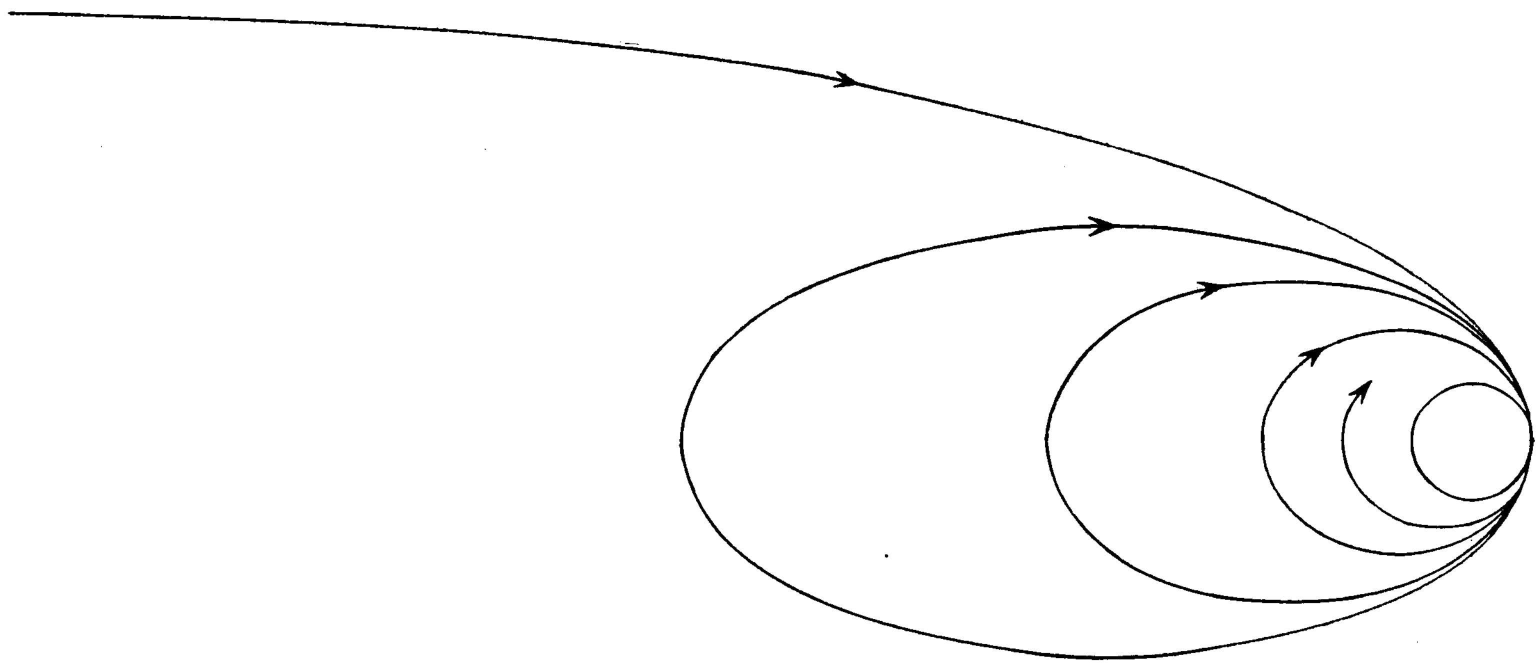


Figure 1.- A schematic diagram of a braking ellipse trajectory.

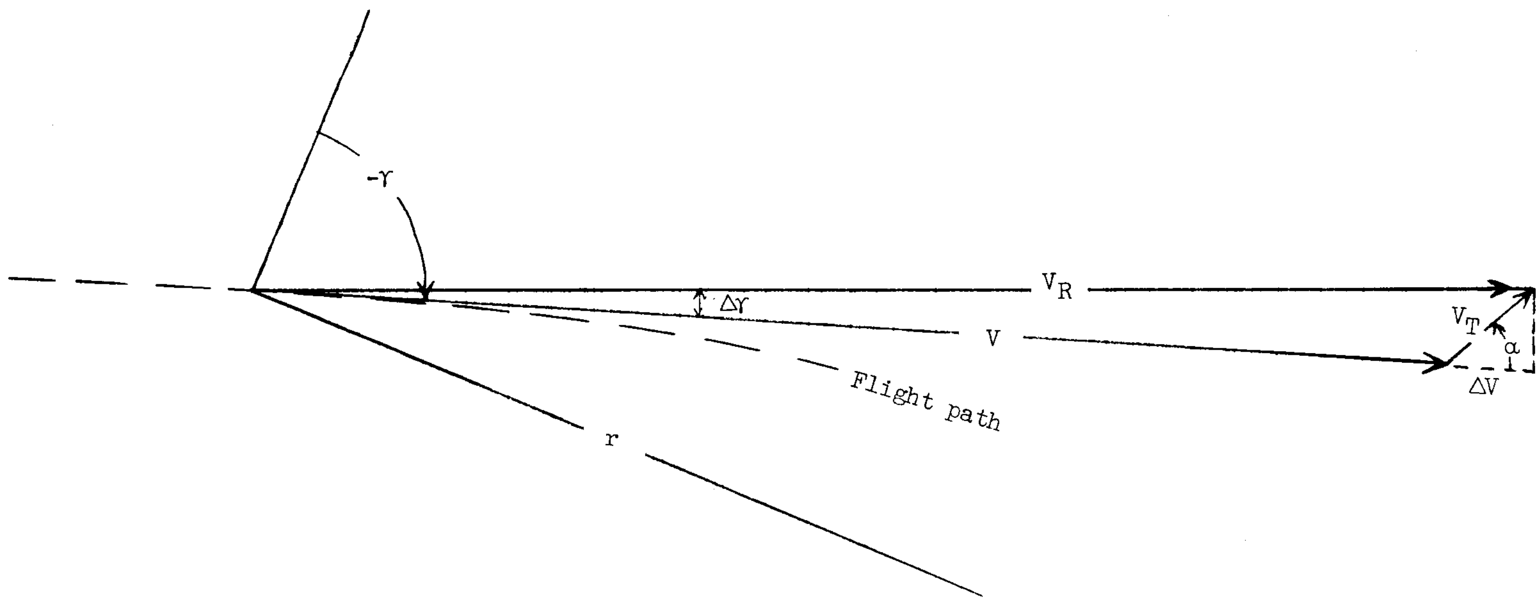
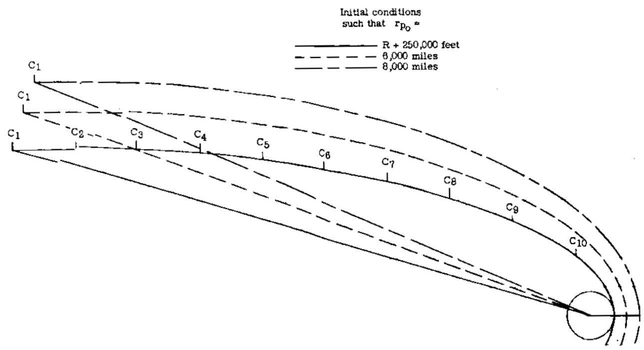
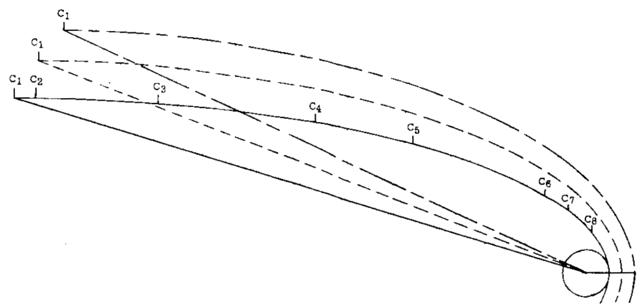


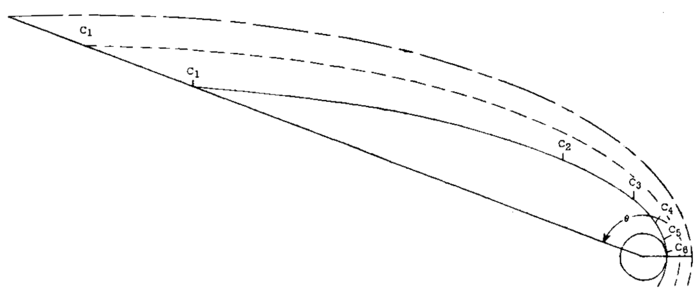
Figure 2.- A geometric diagram of the space vehicle's velocity vector and the corrective velocity vector.



(a) Radial distance method where r_o = 100,000 miles and Δr = 10,000 miles.

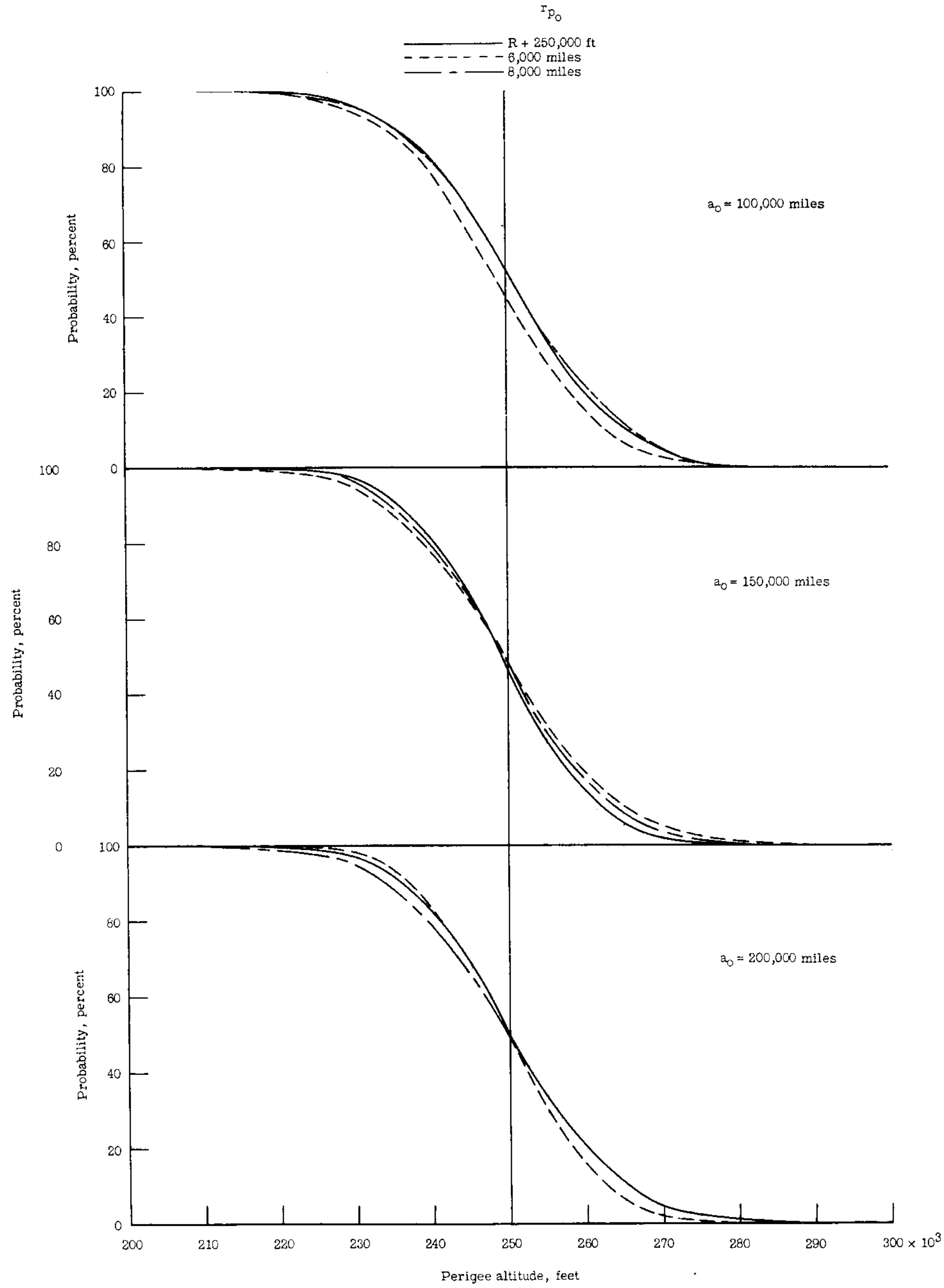


(b) Time method schedule B where t_o = 17 hr. Δt variable.



(c) Angular method where $\theta_o = 160^\circ$ and $\Delta\theta = 30^\circ$.

Figure 3.- Schematic diagrams of three trajectories defined by the initial conditions where r_{p_o} = R + 250,000 feet, 6,000 miles, and 8,000 miles showing (1) the initial points of this study and (2) the correction points along the trajectory defined by the initial conditions where r_{p_o} = R + 250,000 feet for the three methods of scheduling corrective thrust.



(a) Radial distance method.

Figure 4.- Probability of exceeding a given perigee altitude for three values of a_o where the errors in V, γ, and V_T were: $\sigma_V = 1r/10{,}000$ ft/sec, $\sigma_\gamma = 0.0125r/10{,}000$ degree, and $\sigma_{V_T} = 1.3$ percent.

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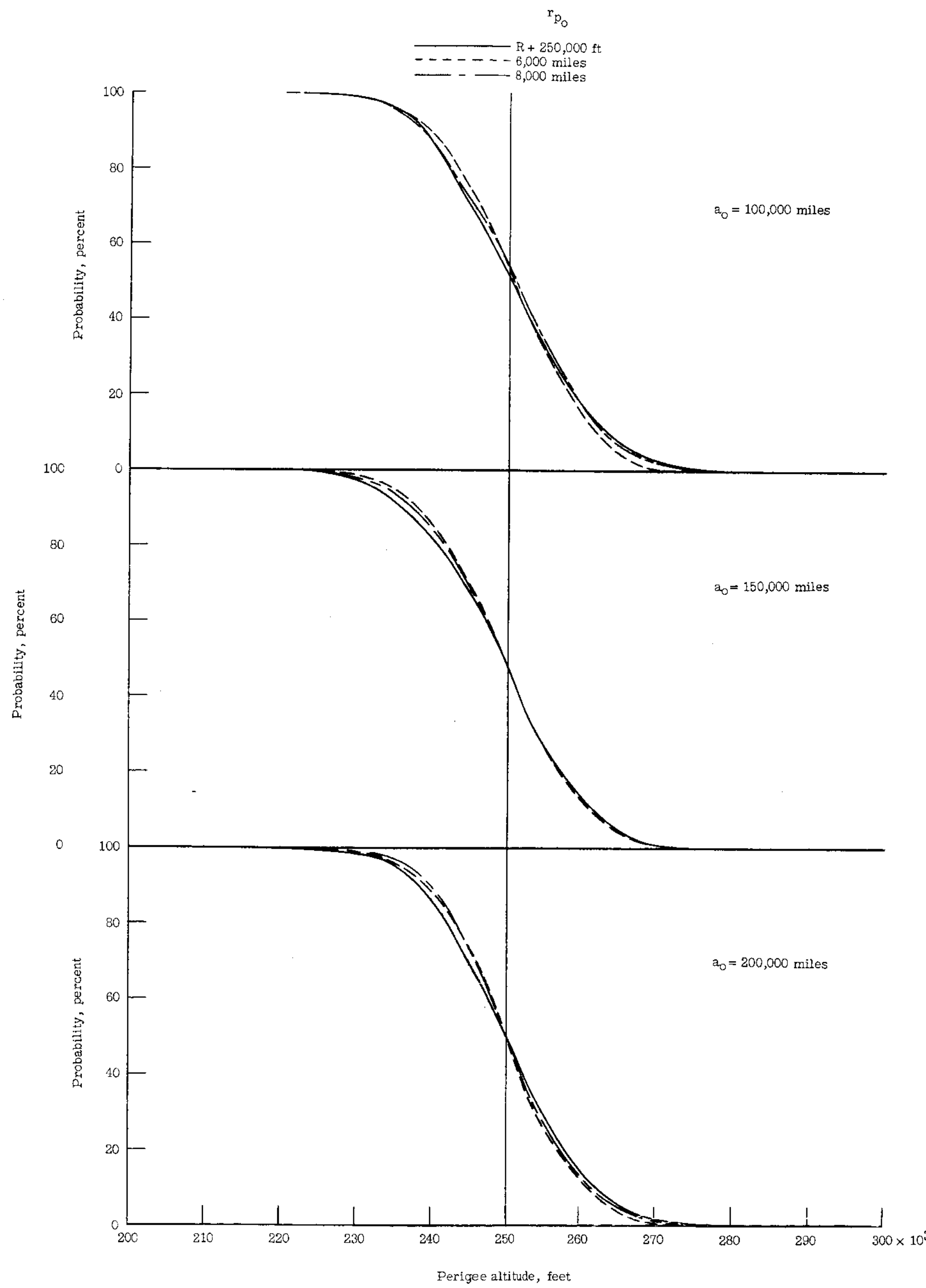
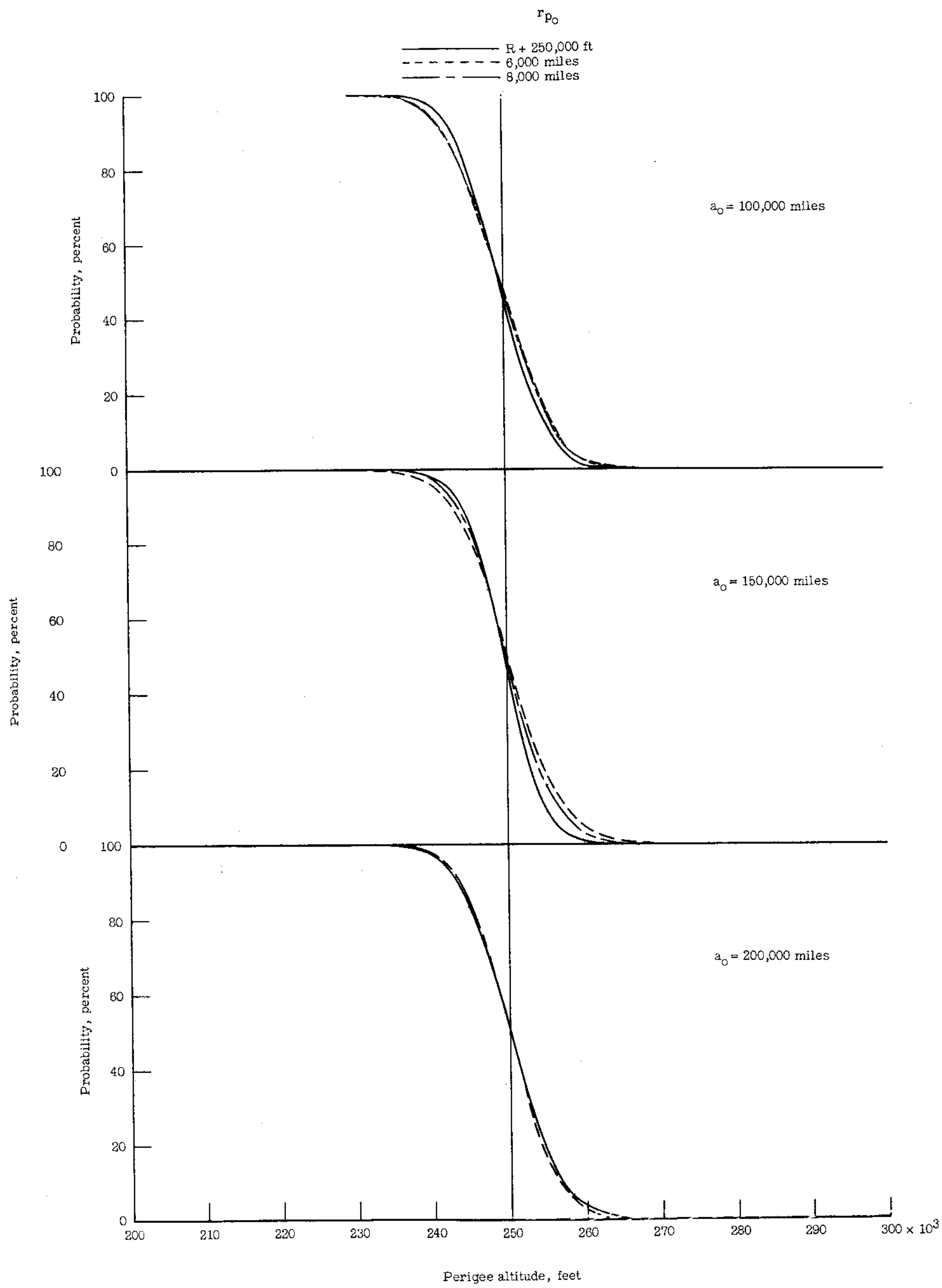


(b) Time method schedule A.

Figure 4.- Continued.



(c) Time method schedule B.

Figure 4.- Continued.

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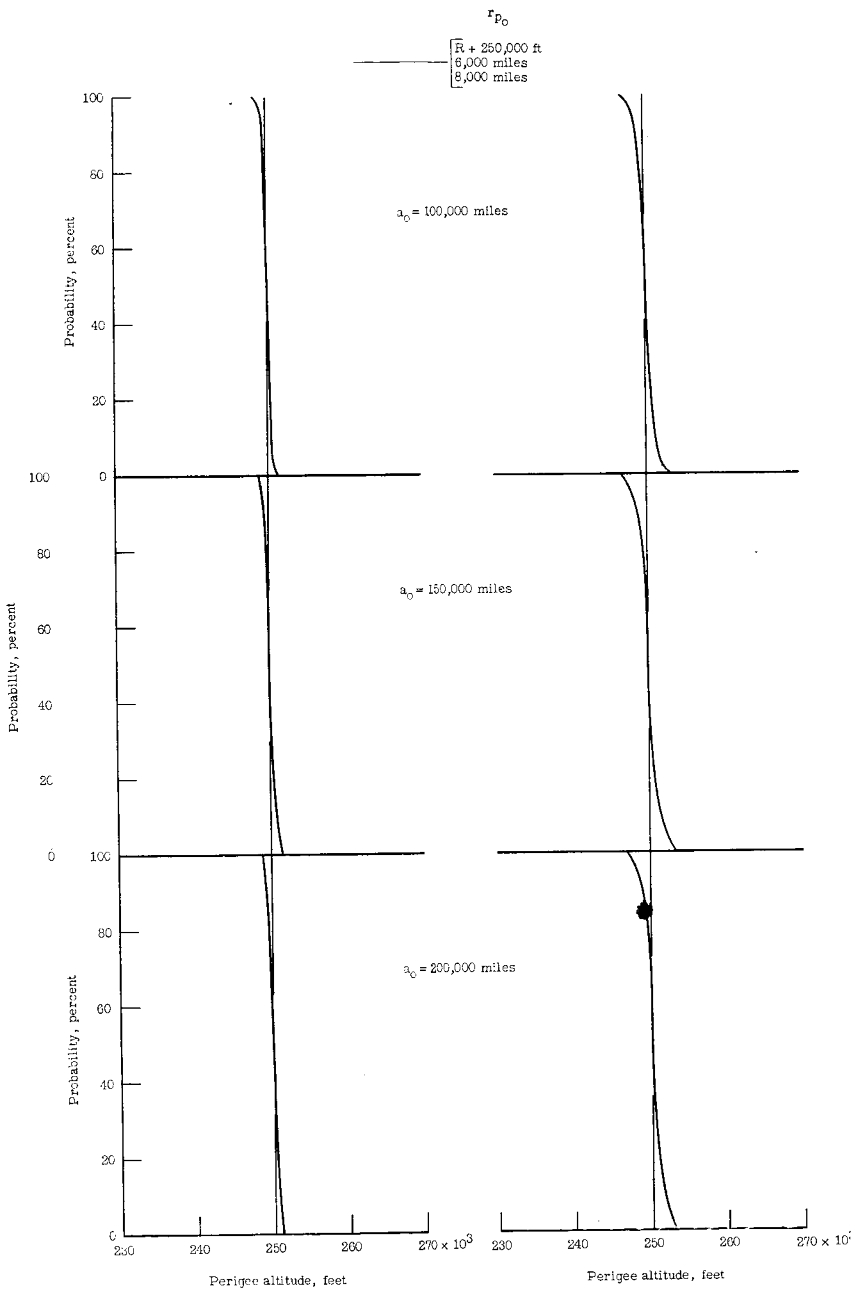
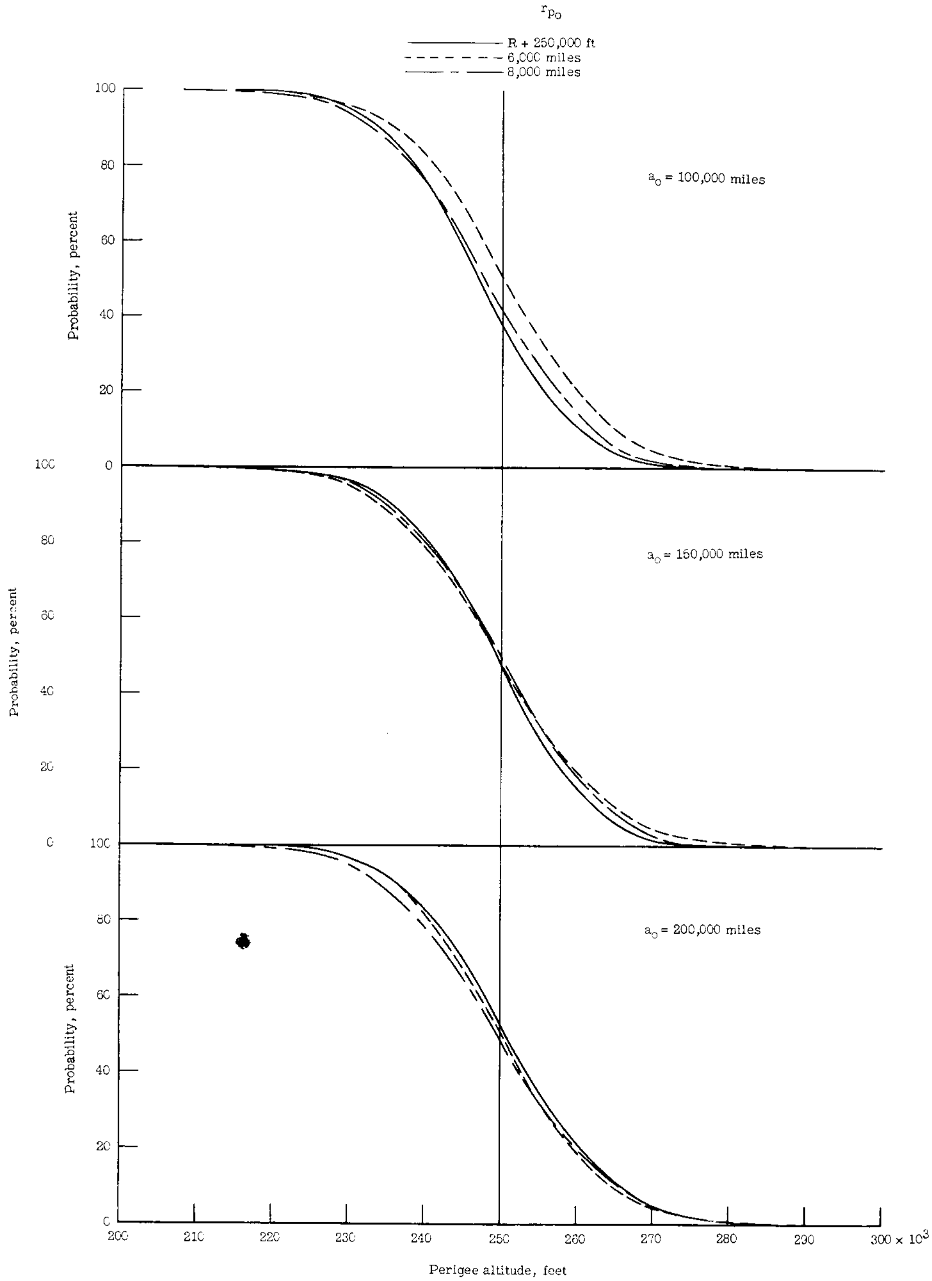


(d) Angular method ($\Delta\theta$ = 30°). (e) Angular method ($\Delta\theta$ = 45°).

Figure 4.- Concluded.



(a) Radial distance method.

Figure 5.- Probability of exceeding a given perigee altitude for three values of a_o where the errors in V, γ, and V_T were: $\sigma_V = 1$ ft/sec, $\sigma_\gamma = 0.0125^\circ$, and $\sigma_{V_T} = 1.3$ percent.

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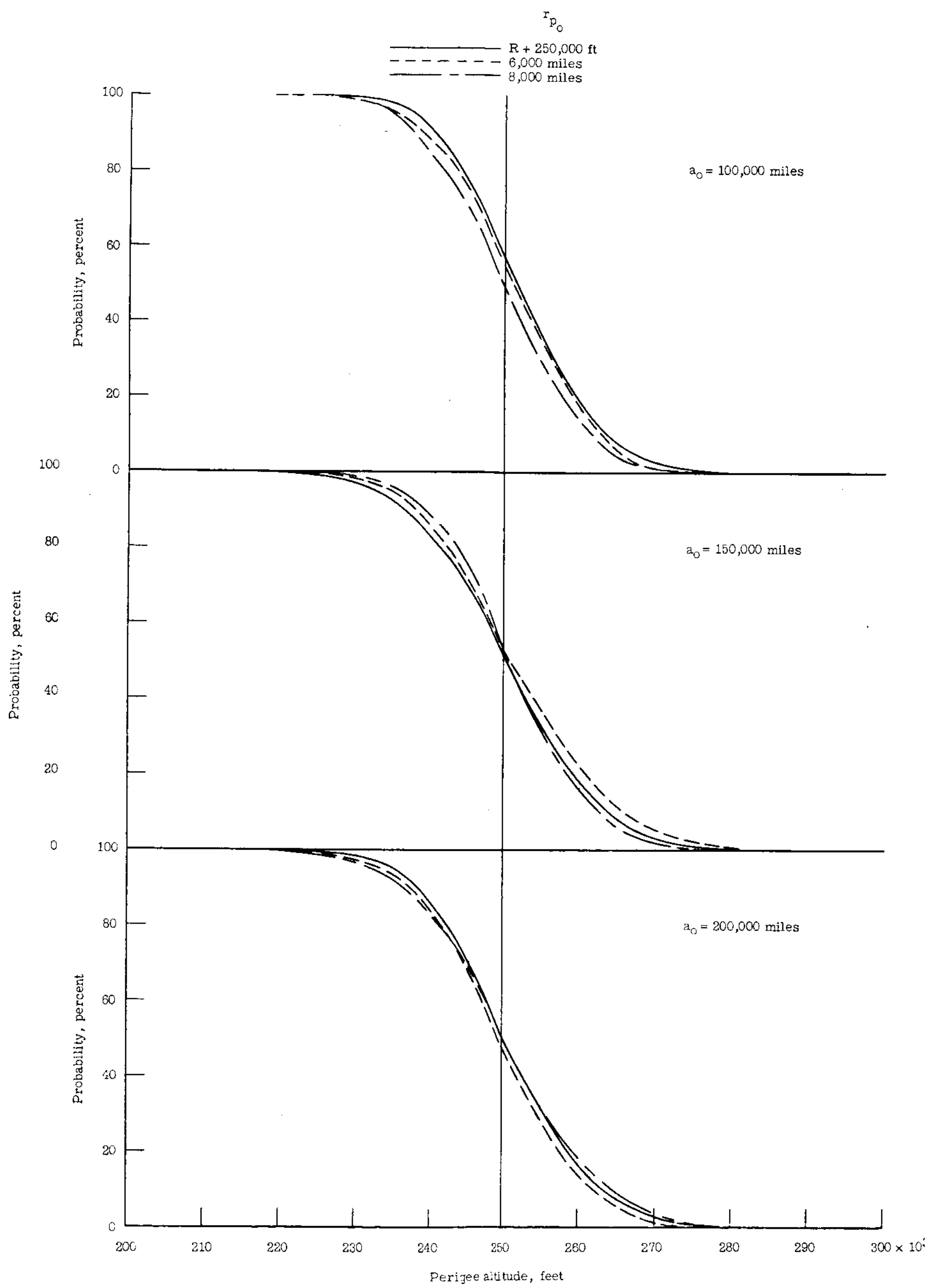
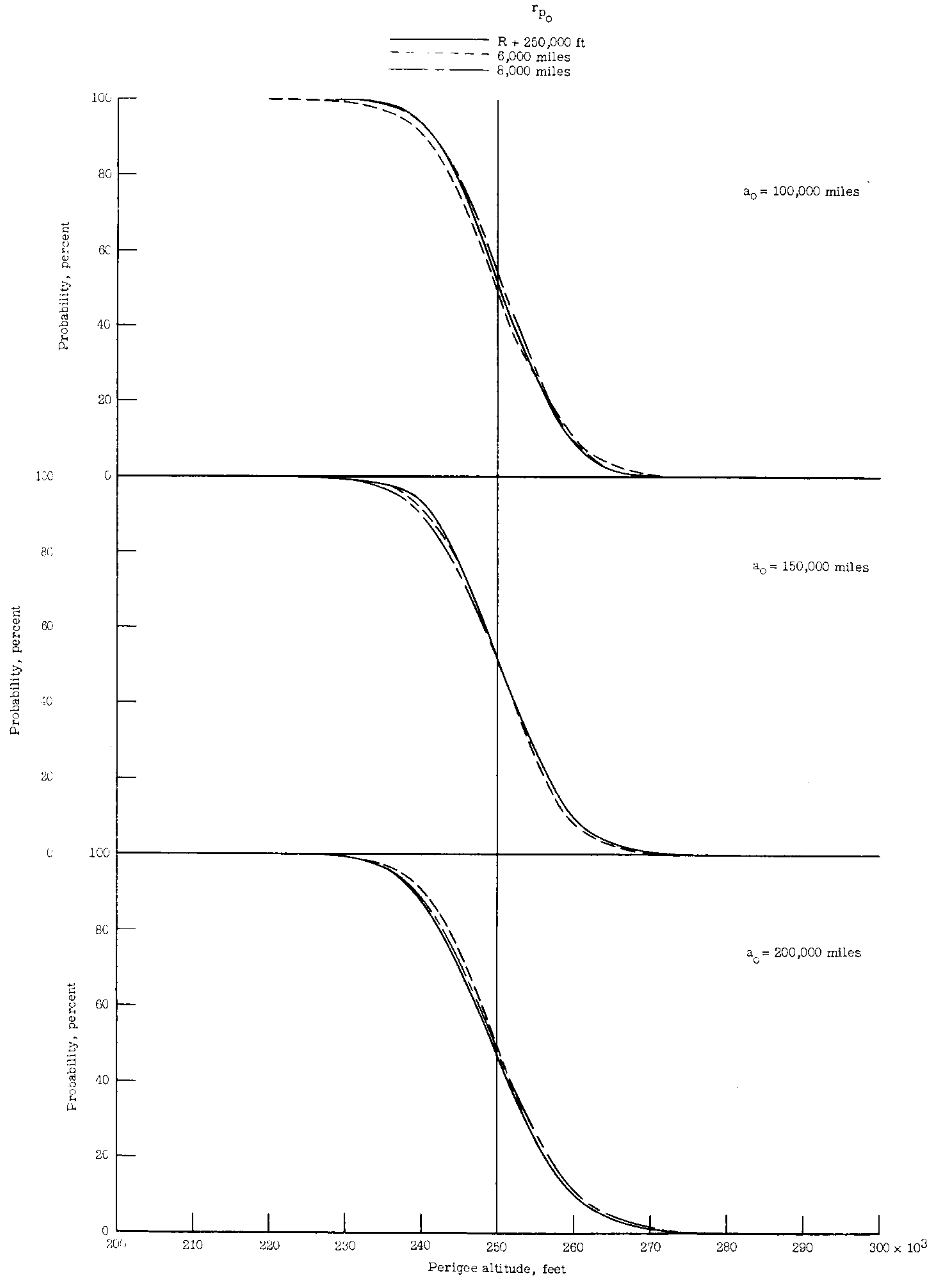


(b) Time method schedule A.

Figure 5.- Continued.



(c) Time method schedule B.

Figure 5.- Continued.

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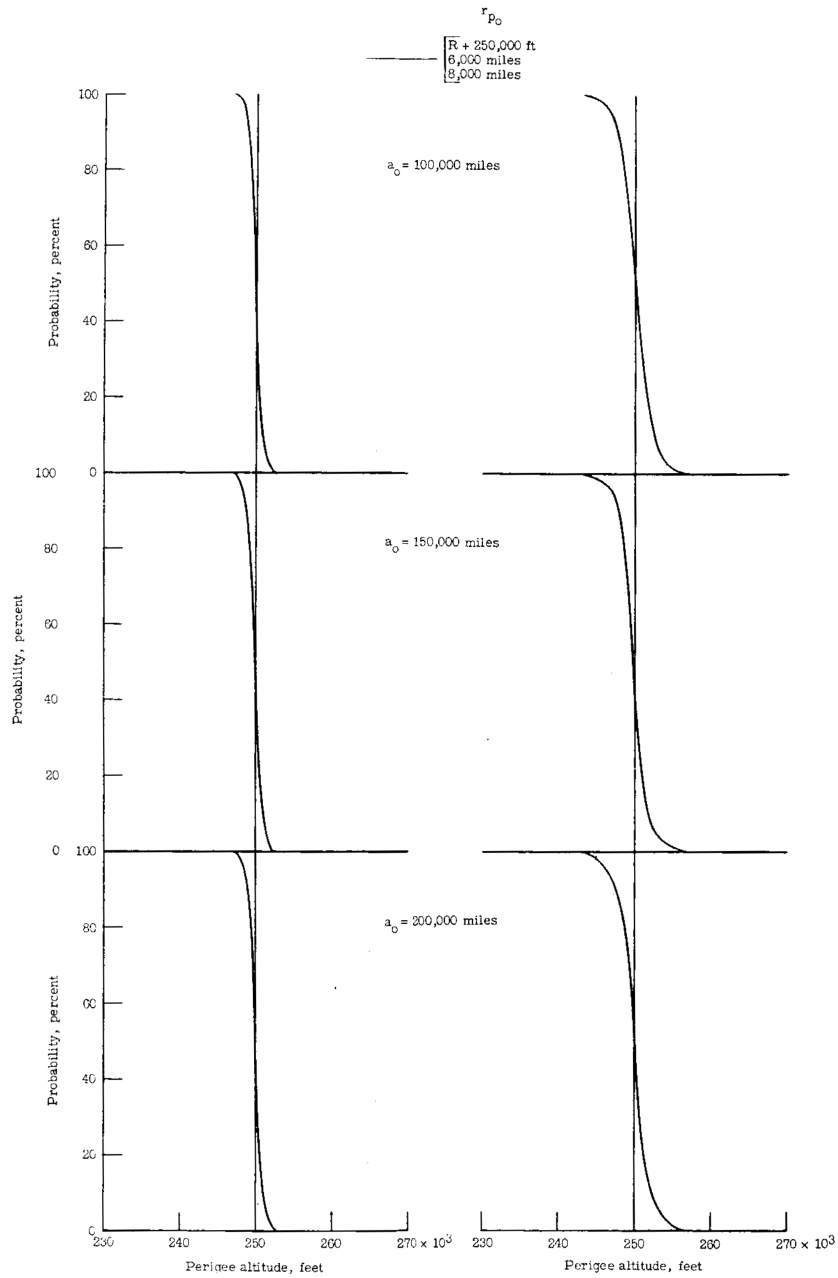

(d) Angular method ($\Delta\theta$ = 30°). (e) Angular method ($\Delta\theta$ = 45°).

Figure 5.- Concluded.

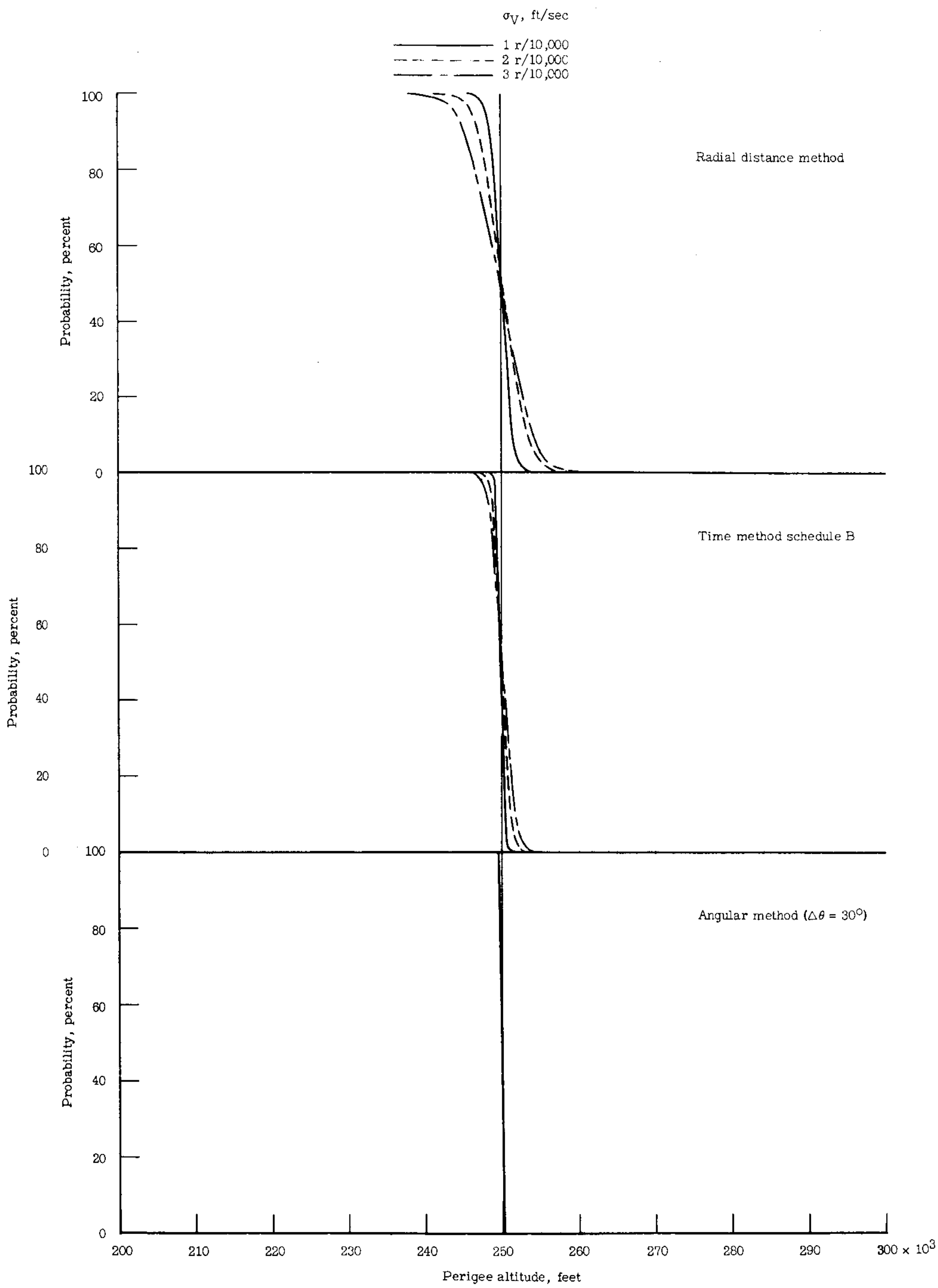

Figure 6.- Probability of exceeding a given perigee altitude for three errors in V that were dependent upon range and where the errors in γ and V_T were zero. a_o = 100,000 miles; r_{p_o} = R + 250,000 feet.

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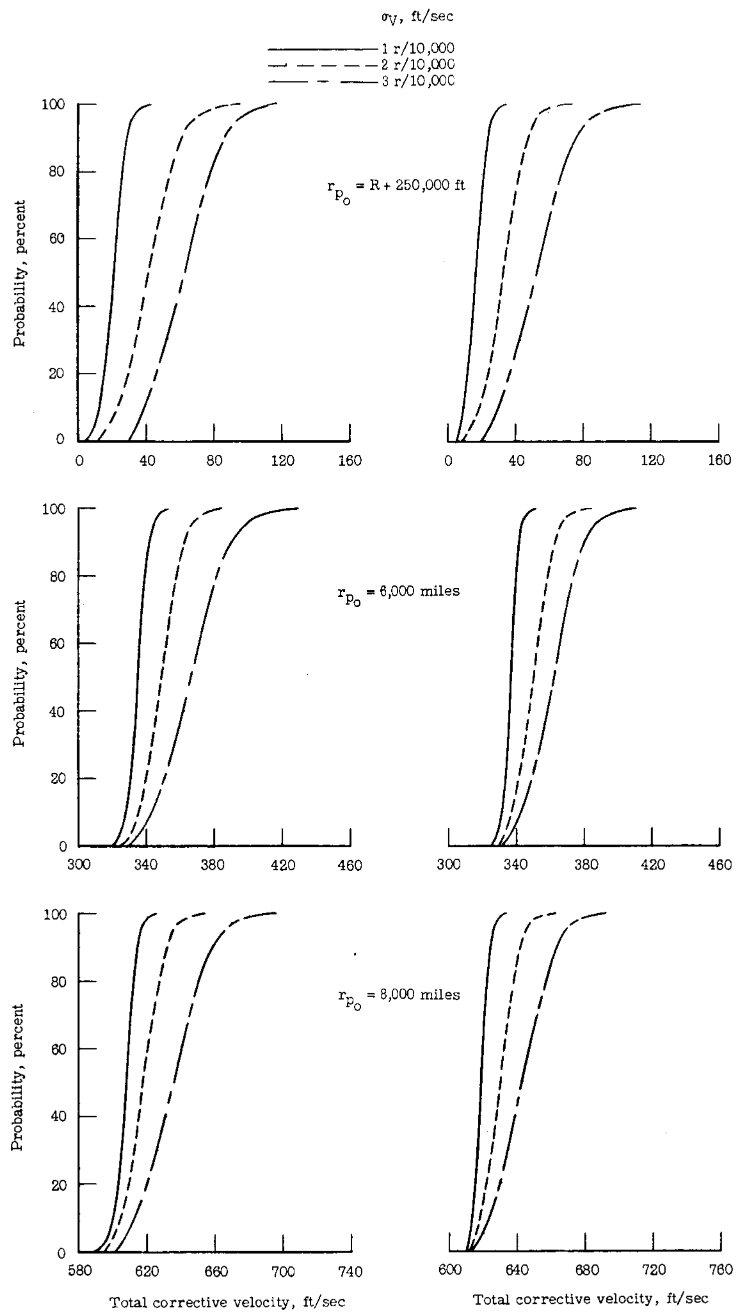

(a) Radial distance method. (b) Time method schedule B.

Figure 7.- Total corrective velocity probability curves for three errors in V that were dependent upon range and where the errors in γ and V_T were zero. a_o = 100,000 miles.

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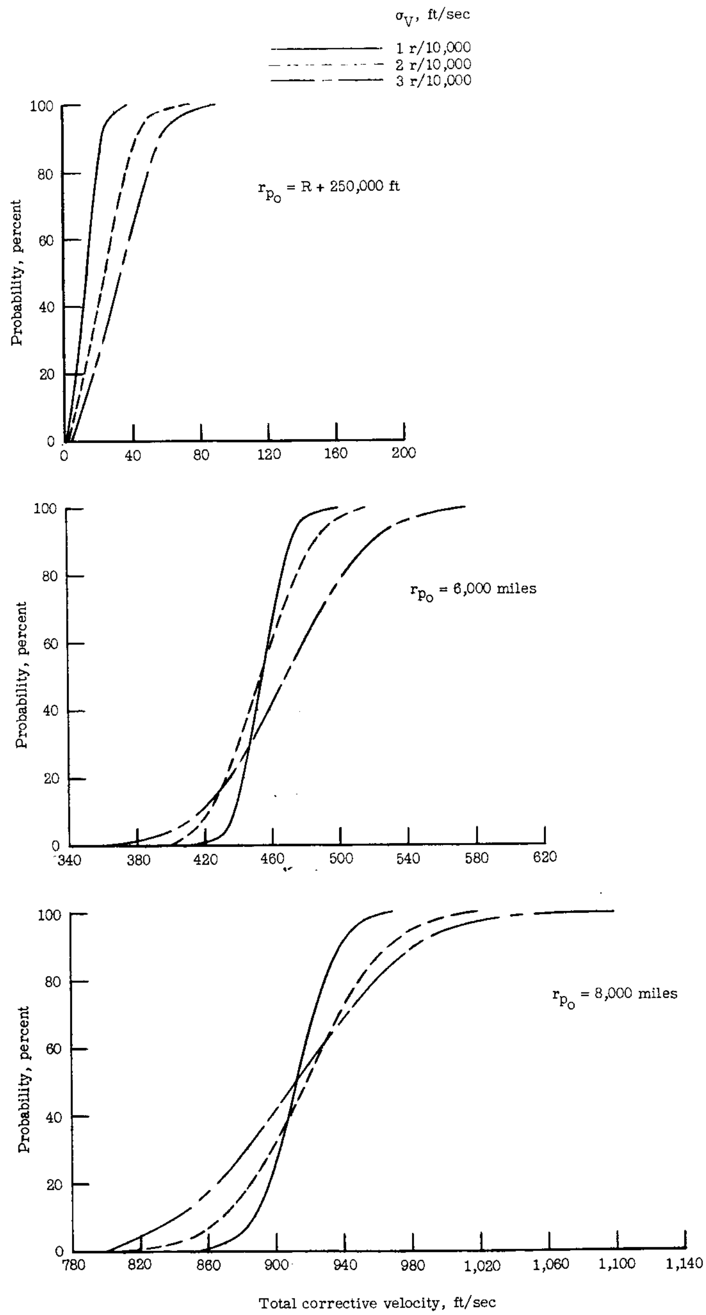

(c) Angular method ($\Delta\theta$ = 30°).

Figure 7.- Concluded.

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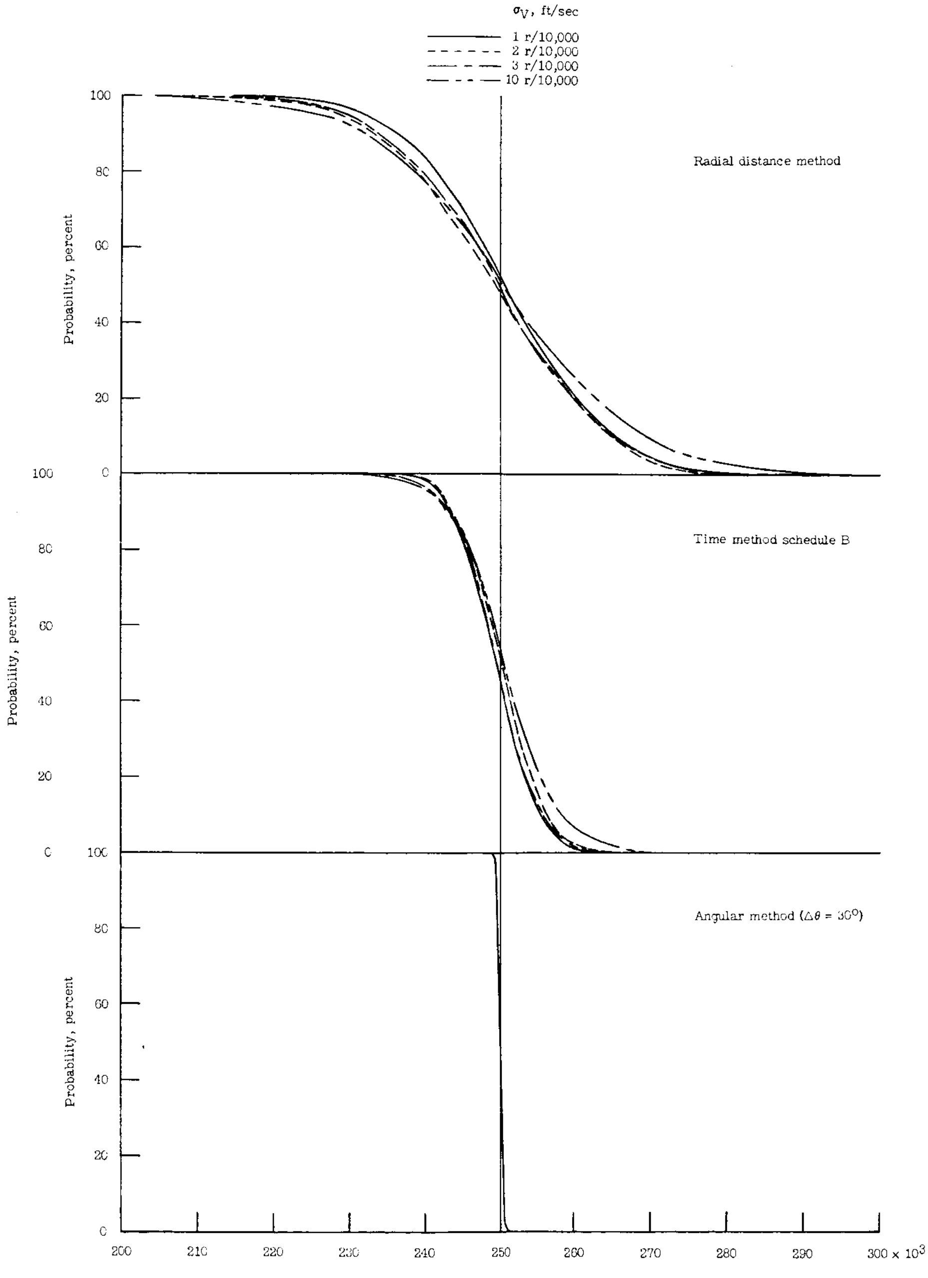

Figure 8.- Probability of exceeding a given perigee altitude for four errors in V that were dependent upon range and where the errors in γ and V_T were $\sigma_\gamma = 0.0125r/10{,}000$ degree and $\sigma_{V_T} = 1.3$ percent. $a_o = 100{,}000$ miles; $r_{p_o} = R + 250{,}000$ feet.

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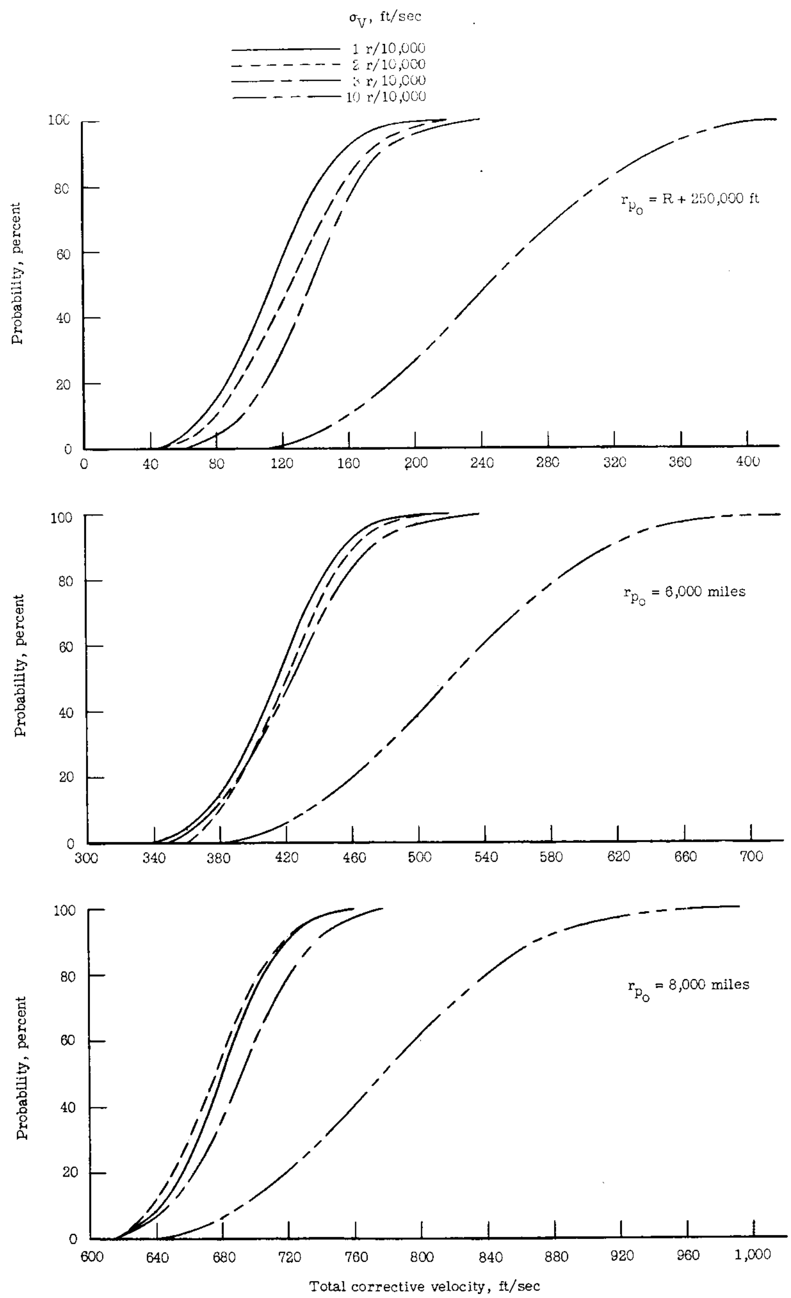

(a) Radial distance method.

Figure 9.- Total corrective velocity probability curves for four errors in V that were dependent upon range and where the errors in γ and V_T were $\sigma_\gamma = 0.0125r/10{,}000$ degree and $\sigma_{V_T} = 1.3$ percent.

a_o = 100,000 miles.

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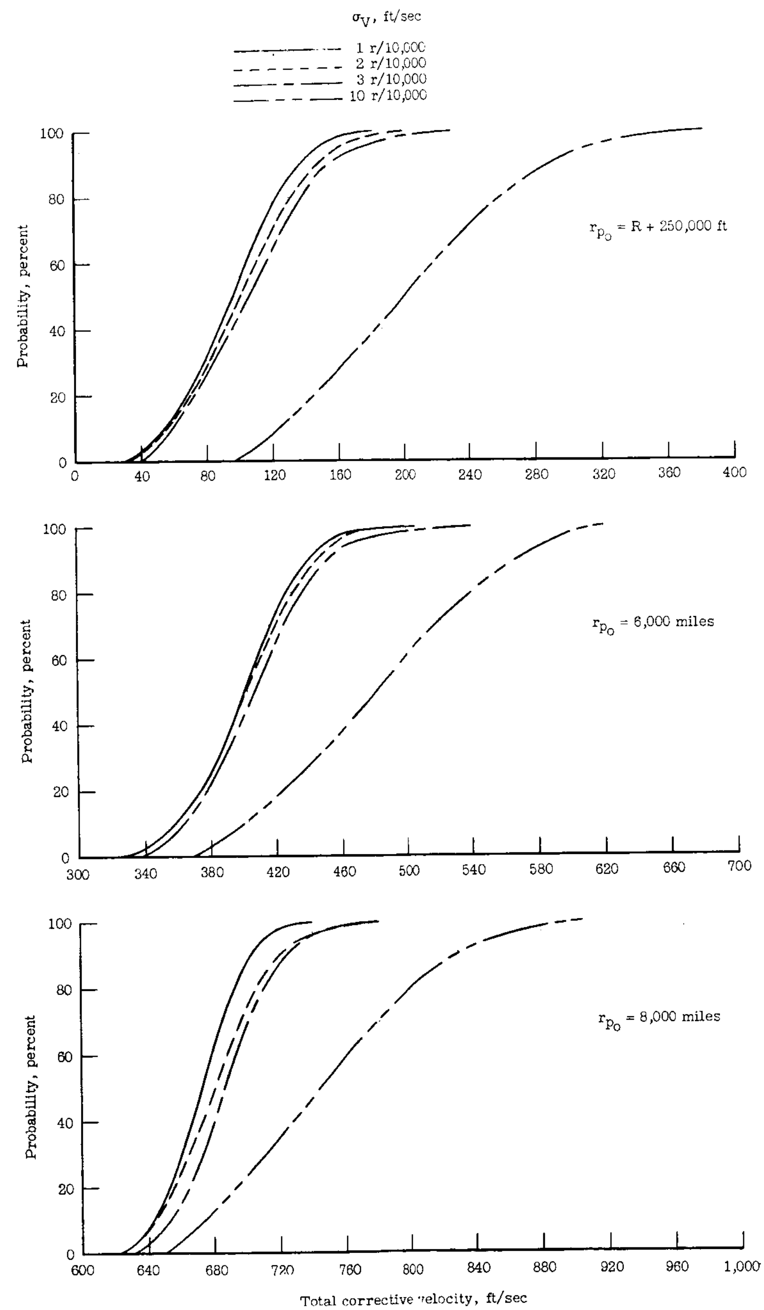
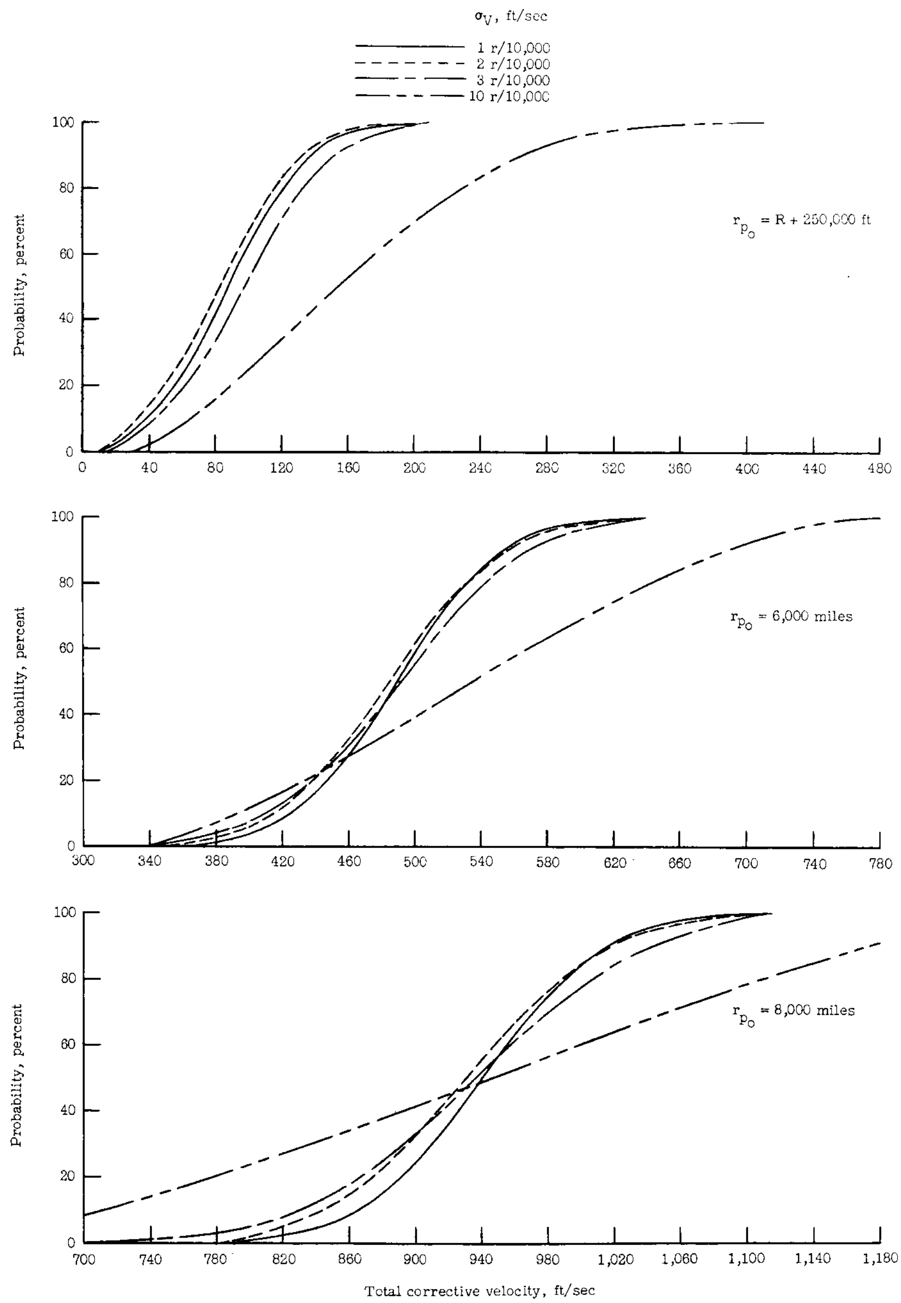


(b) Time method schedule B.

Figure 9.- Continued.



(c) Angular method ($\Delta\theta = 30^{\circ}$).

Figure 9.- Concluded.

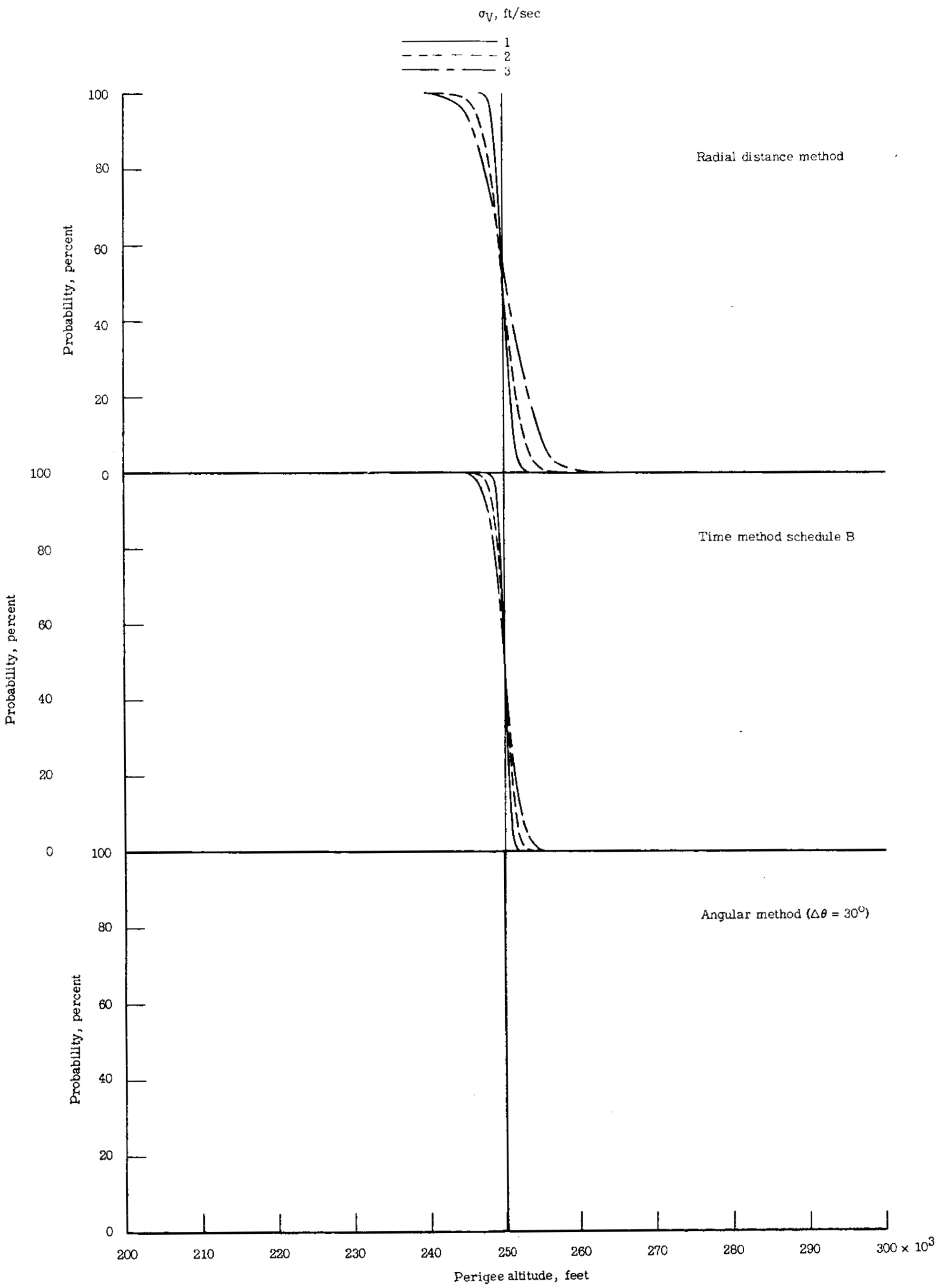
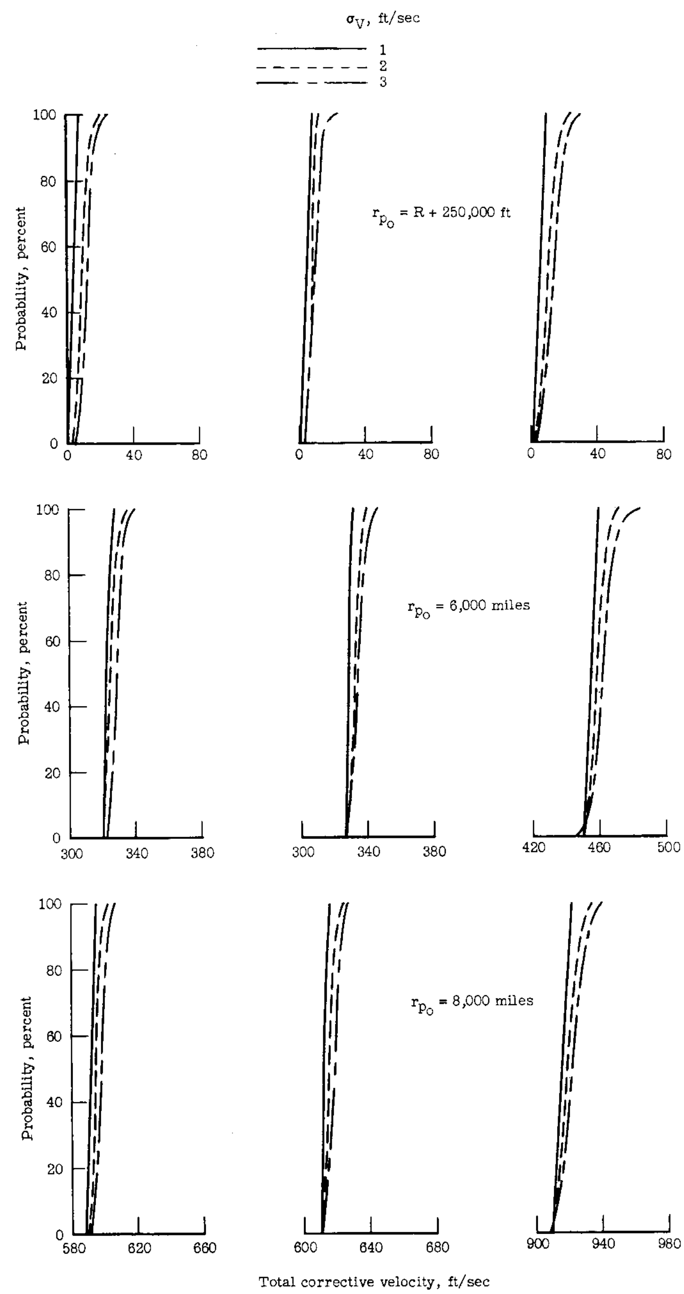


Figure 10.- Probability of exceeding a given perigee altitude for three errors in V that were not dependent upon range and where the errors in γ and V_T were zero. $a_o = 100{,}000$ miles; $r_{p_o} = R + 250{,}000$ feet.



(a) Radial distance method.

(b) Time method schedule B.

(c) Angular method ($\Delta\theta = 30^{\circ}$).

'igure 11.- Total corrective velocity probability curves for three errors in V that were not dependent upon range and where the errors in γ and V_T were zero. a_o = 100,000 miles.

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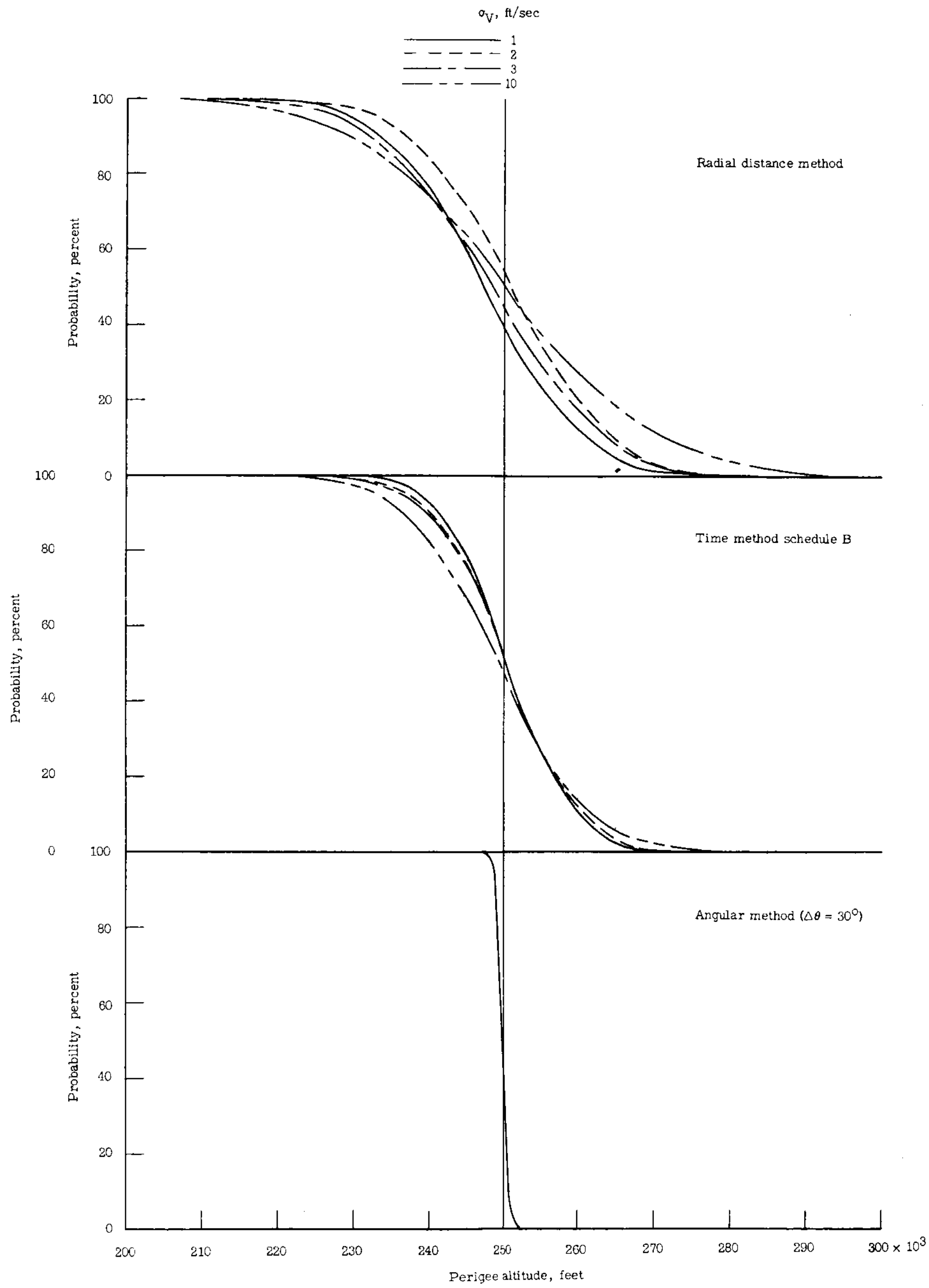
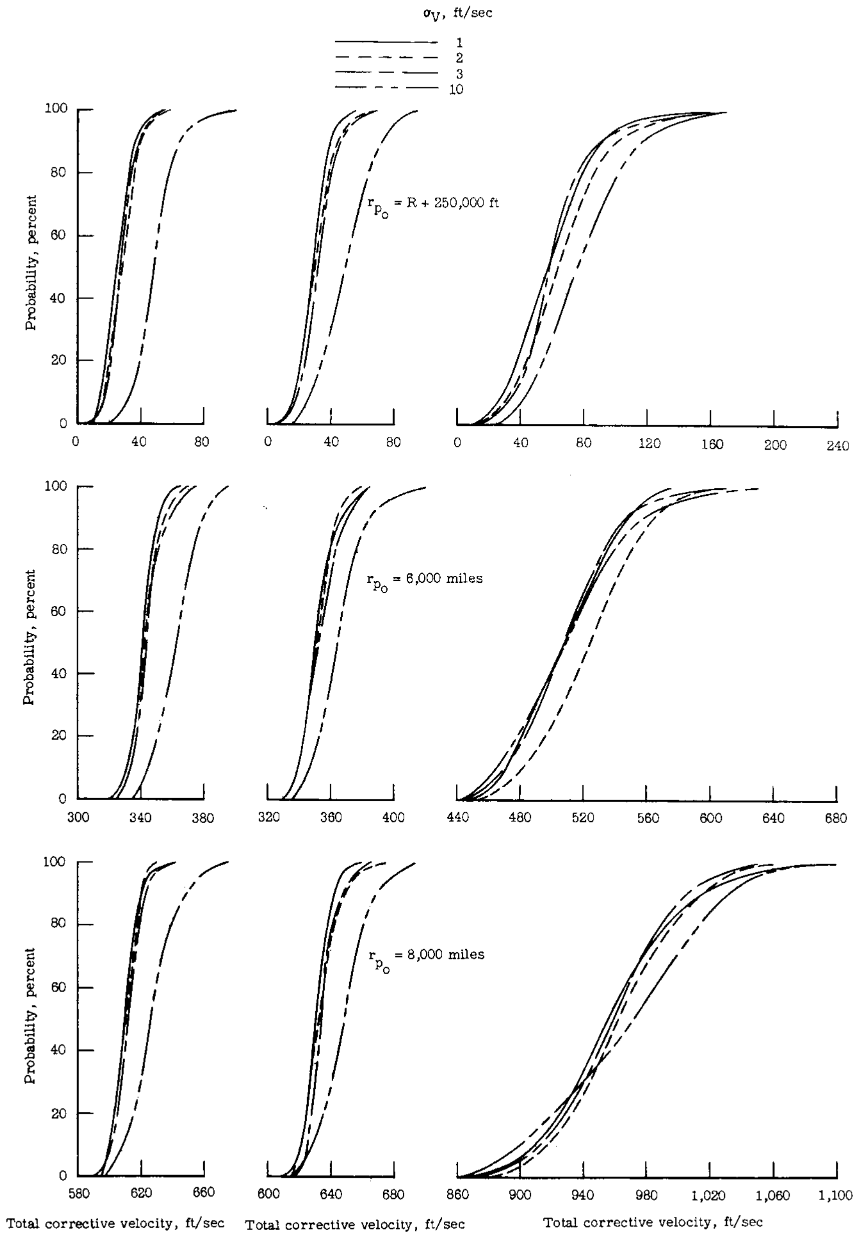


Figure 12.- Probability of exceeding a given perigee altitude for four errors in V that were not dependent upon range and where the error: in γ and V_T were $\sigma_\gamma = 0.0125^{\circ}$ and $\sigma_{V_T} = 1.3$ percent. $a_o = 100{,}000$ miles; $r_{p_o} = R + 250{,}000$ feet.



(a) Radial distance method.

(b) Time method schedule B.

(c) Angular method ($\Delta\theta$ = 30°).

Figure 13.- Total corrective velocity probability curves for four errors in V that were not dependent upon range and where the errors in γ and V_T were $\sigma_\gamma = 0.0125^\circ$ and $\sigma_{V_T} = 1.3$ percent.

a_o = 100,000 miles.

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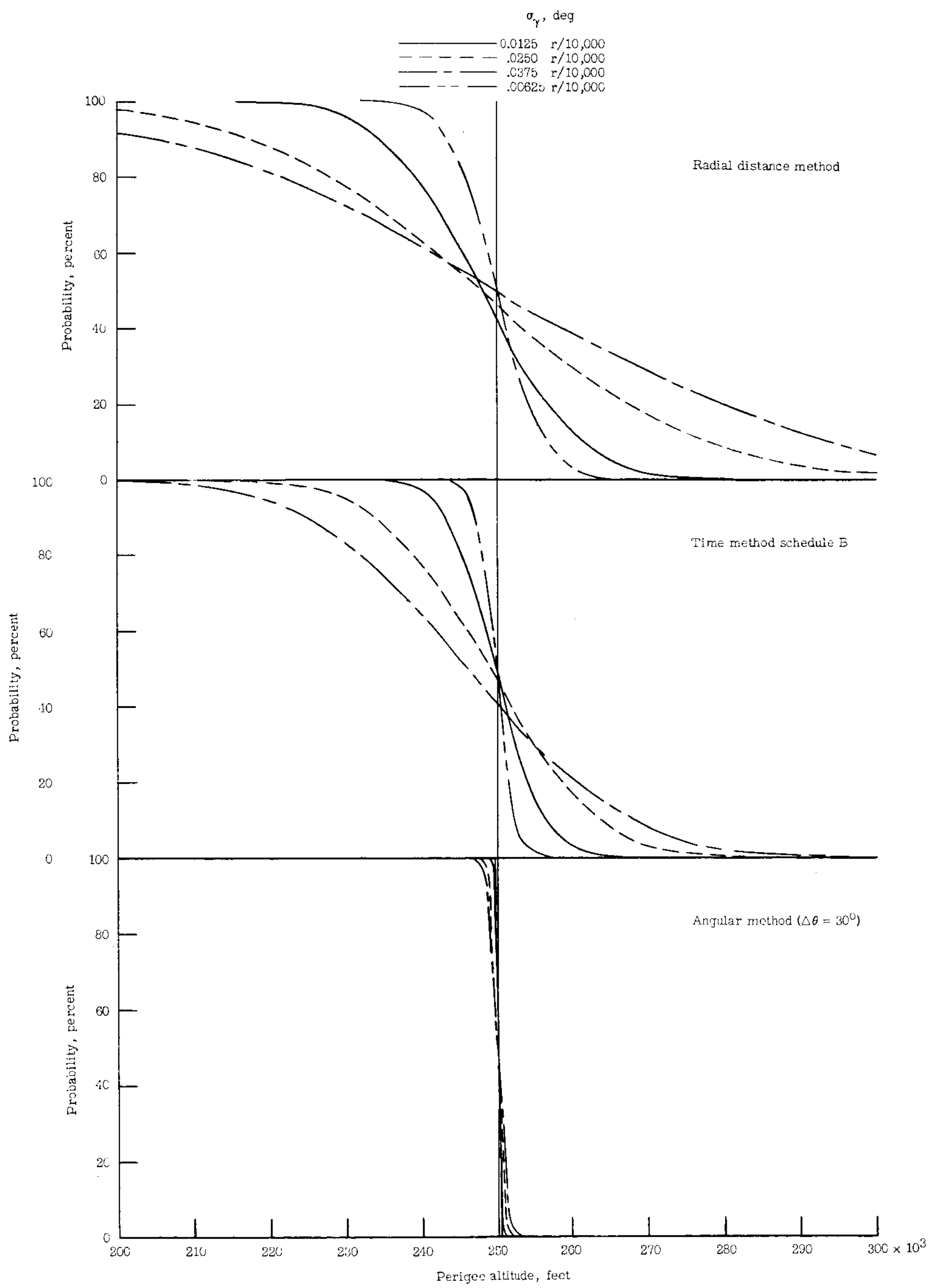

Figure 14.- Probability of exceeding a given perigee altitude for four errors in γ that were dependent upon range and where the errors in V and V_T were zero. $a_o = 100{,}000$ miles; $r_{p_o} = R + 250{,}000$ feet.

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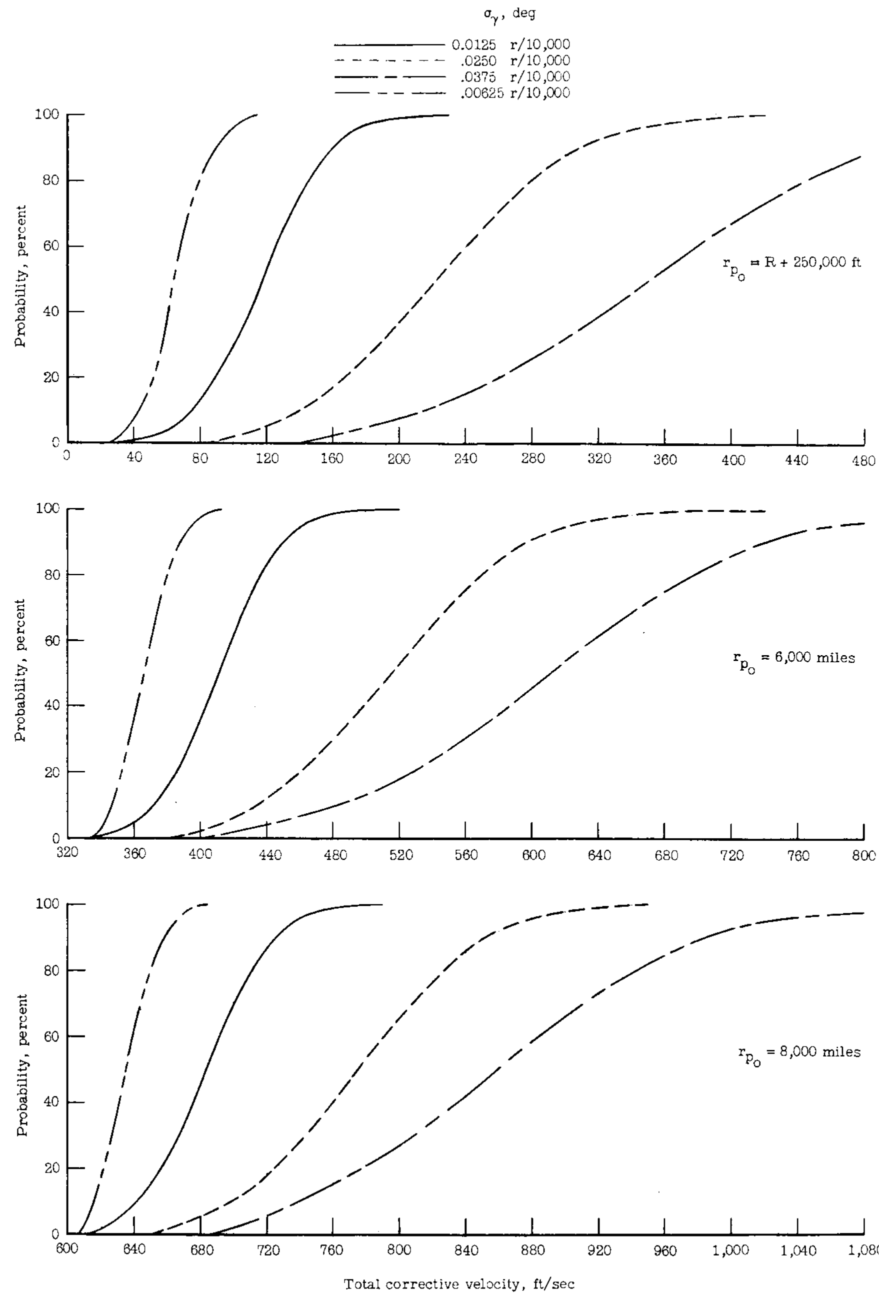
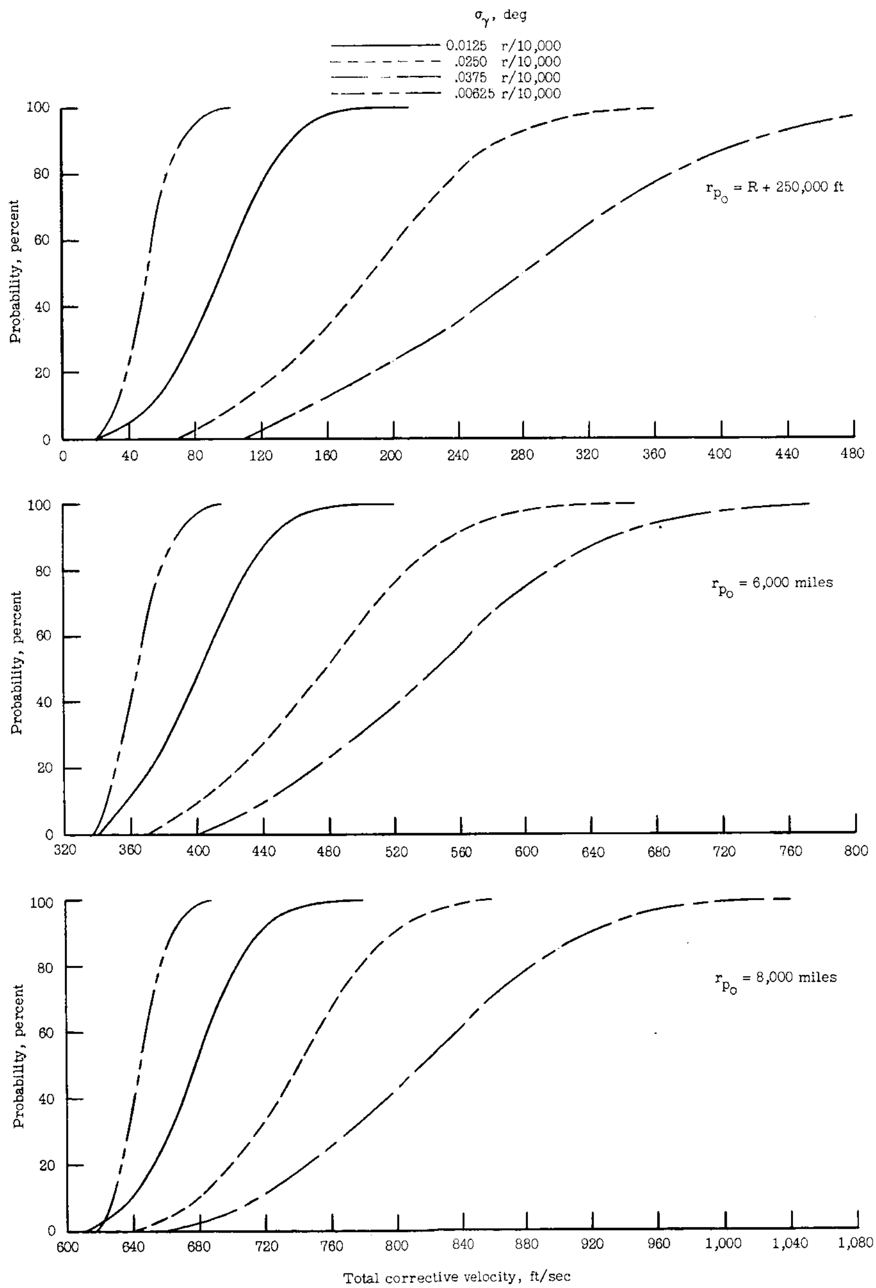


(a) Radial distance method.

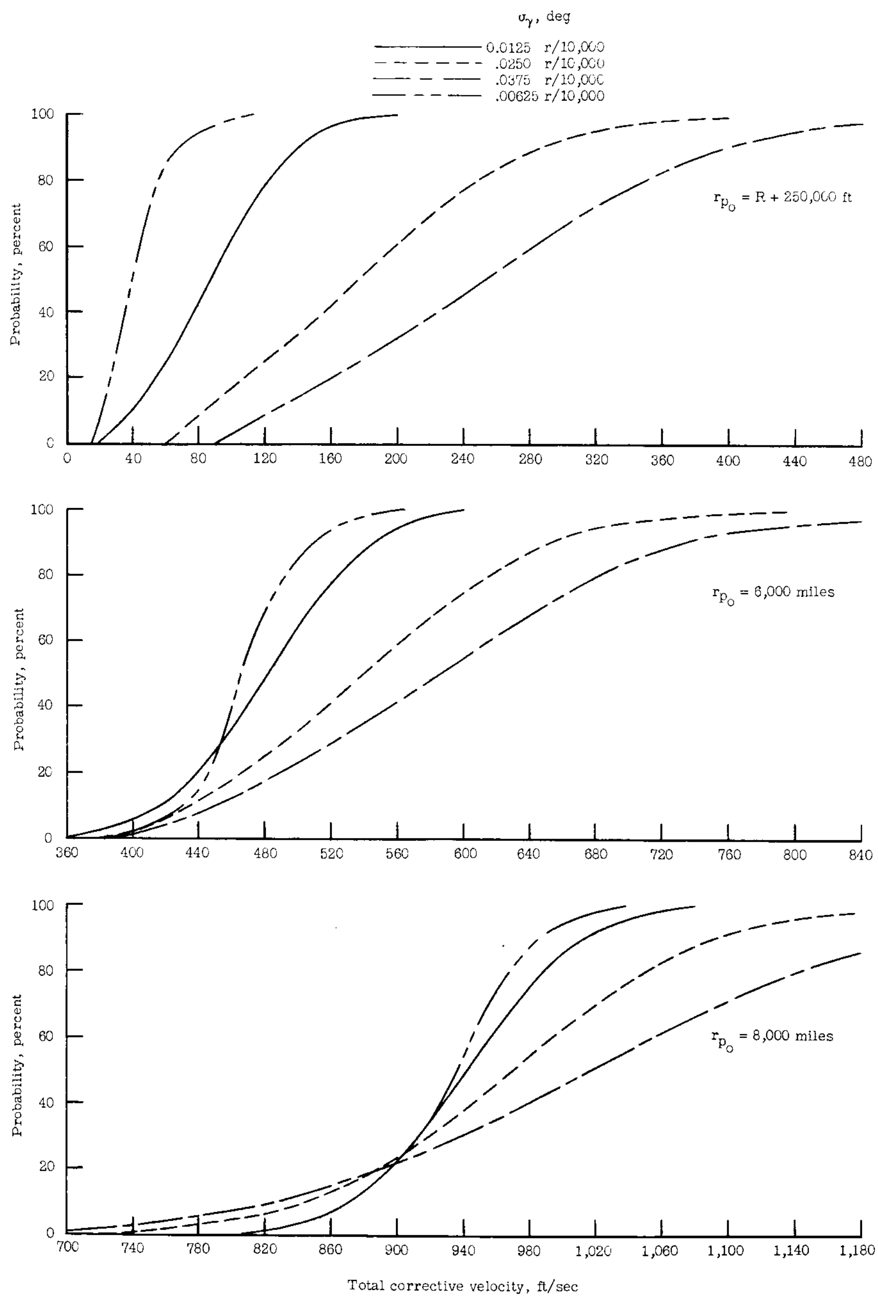
Figure 15.- Total corrective velocity probability curves for four errors in γ that were dependent upon range and where the errors in V and V_T were zero. a_o = 100,000 miles.

L-702



(b) Time method schedule B.

Figure 15.- Continued.



(c) Angular method ($\Delta\theta = 30^\circ$).

Figure 15.- Concluded.

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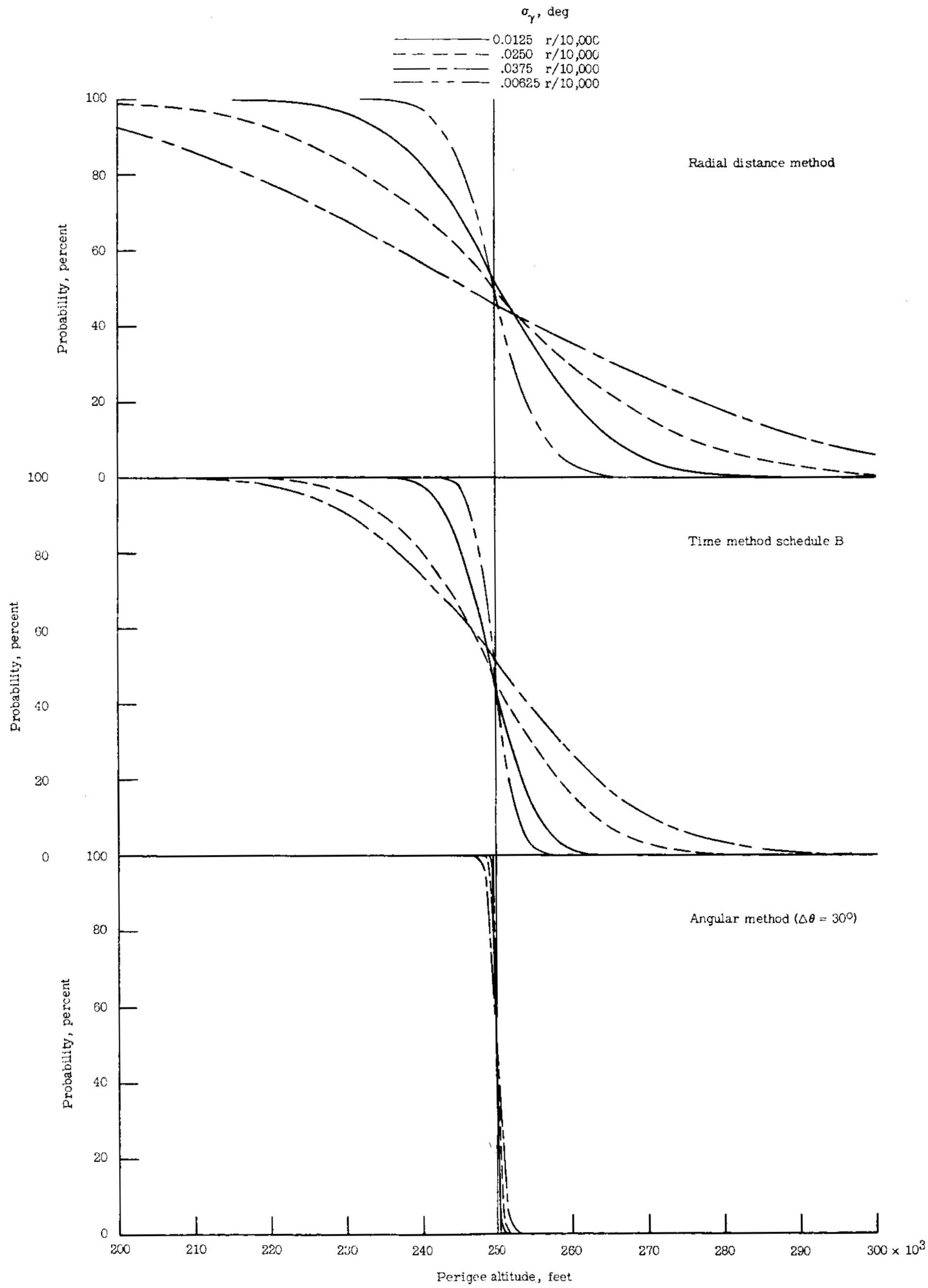
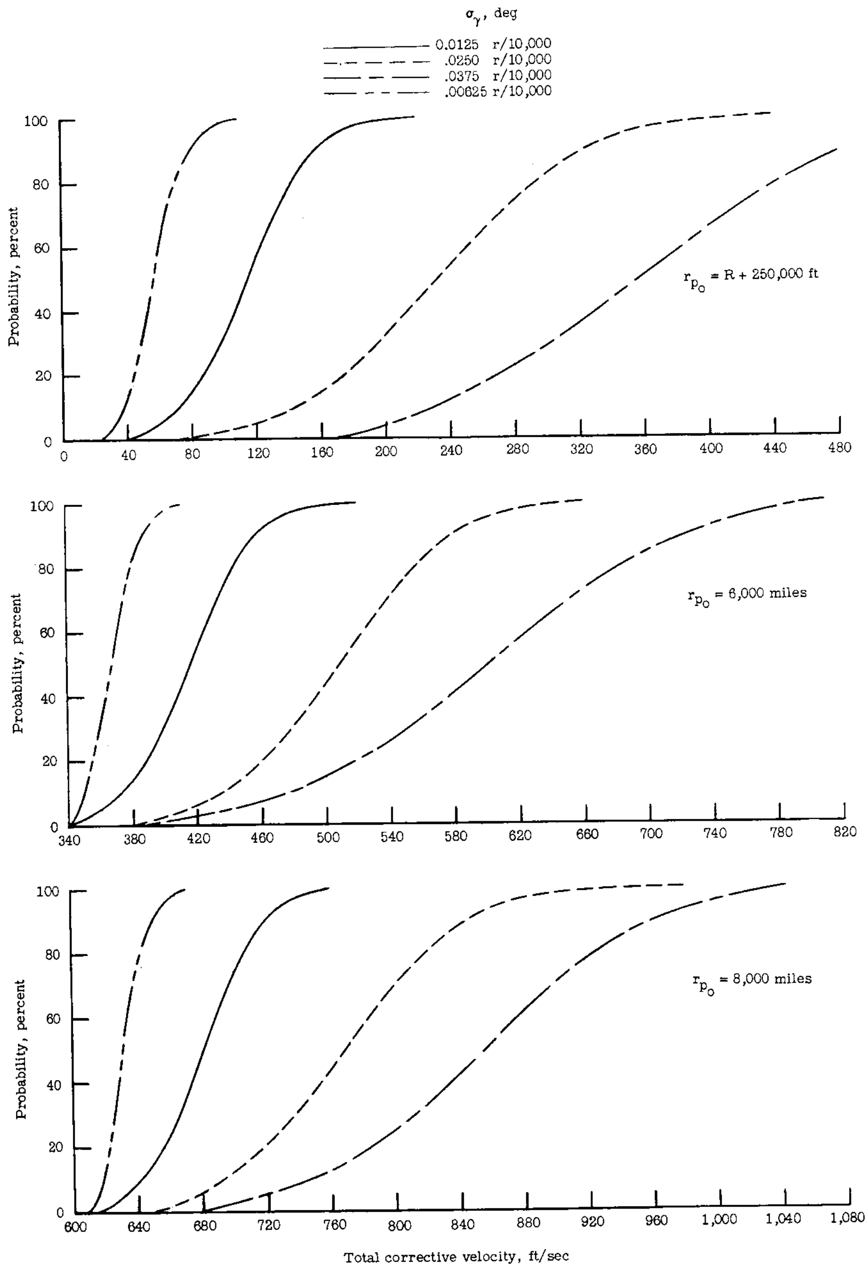


Figure 16.- Probability of exceeding a given perigee altitude for four errors in γ that were dependent upon range and where the errors in V and V_T were $\sigma_V = 1r/10{,}000$ ft/sec and $\sigma_{V_T} = 1.3$ percent. $a_o = 100{,}000$ miles; $r_{p_o} = R + 250{,}000$ miles.



(a) Radial distance method.

Figure 17.- Total corrective velocity probability curves for four errors in γ that were dependent upon range and where the errors in V and V_T were σ_V = 1r/10,000 ft/sec and σ_{V_T} = 1.3 percent.

a_o = 100,000 miles.

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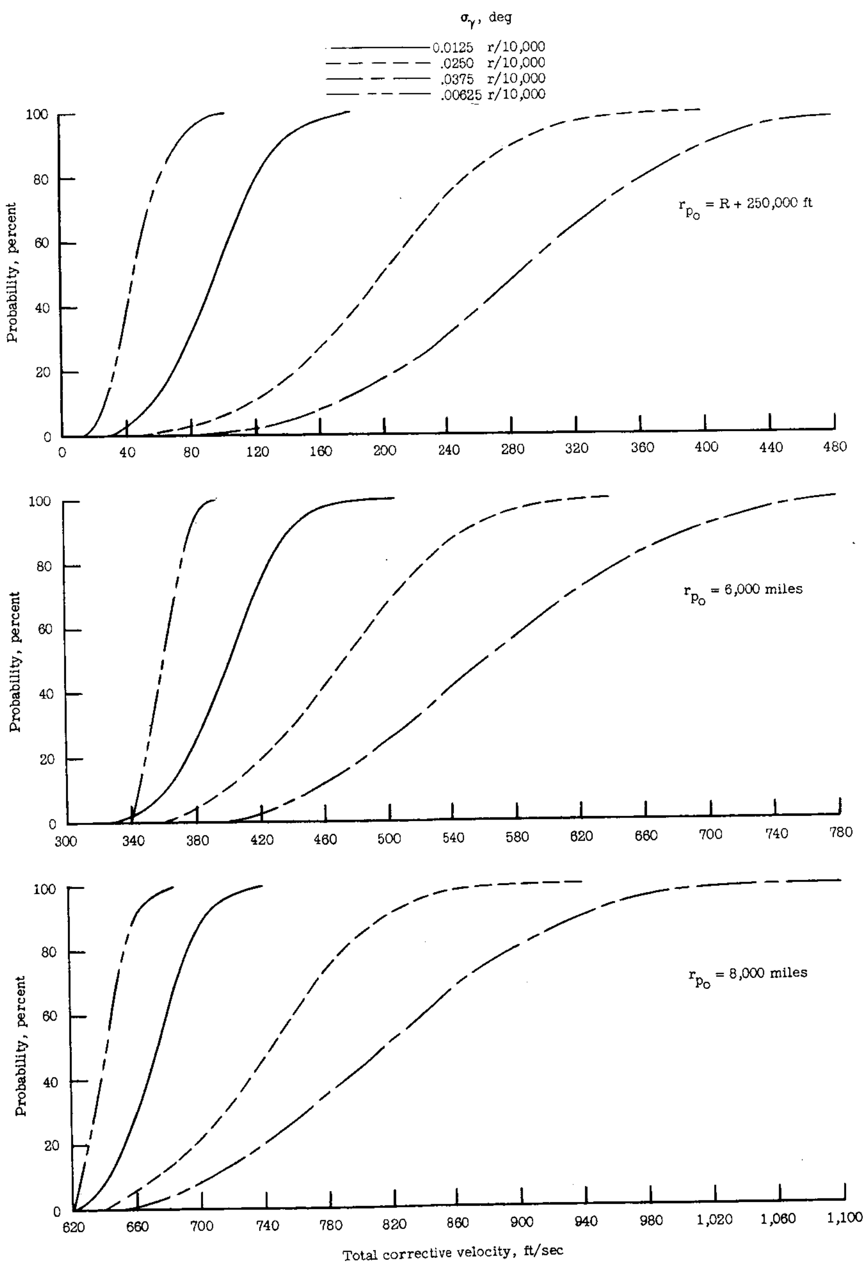
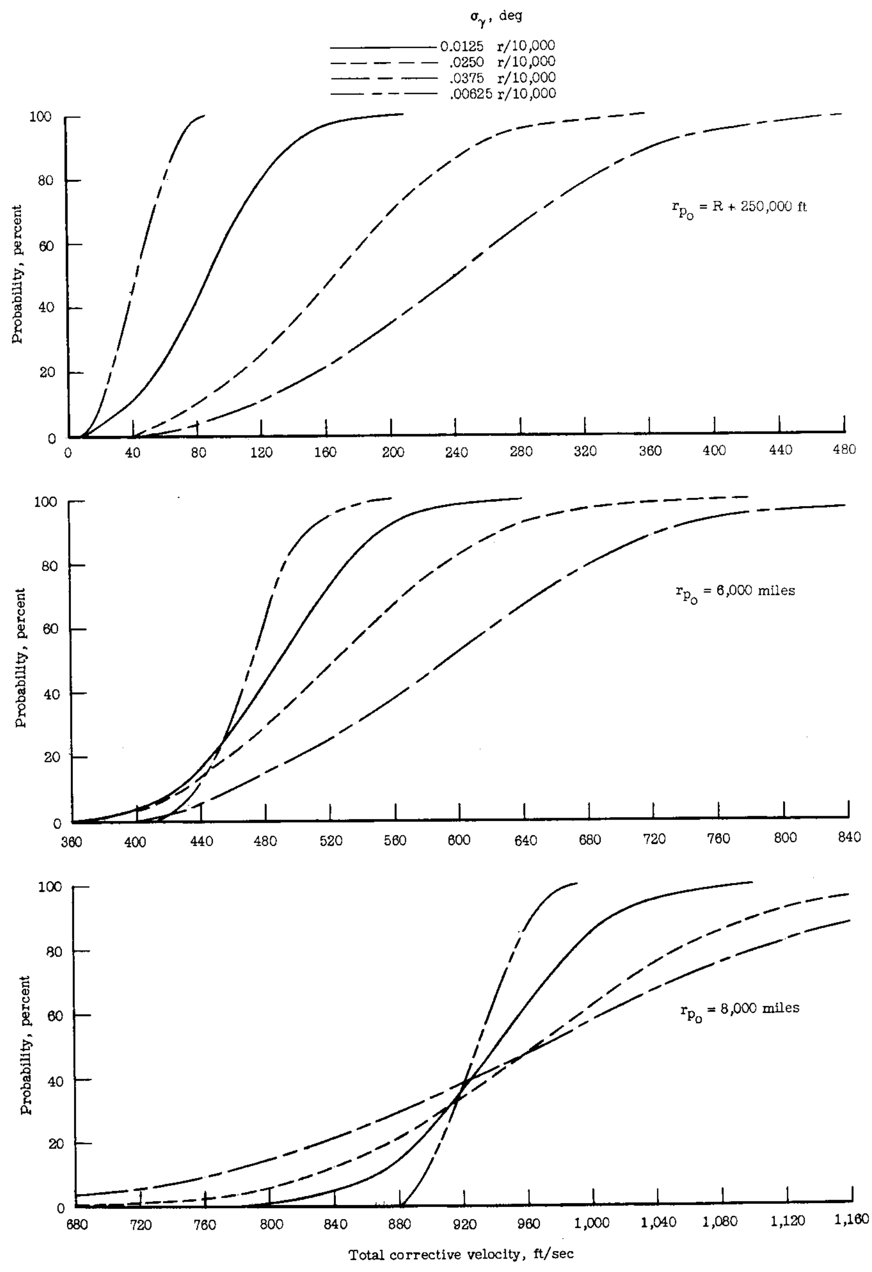


(b) Time method schedule B.

Figure 17.- Continued.



(c) Angular method ($\Delta\theta = 30^\circ$).

Figure 17.- Concluded.

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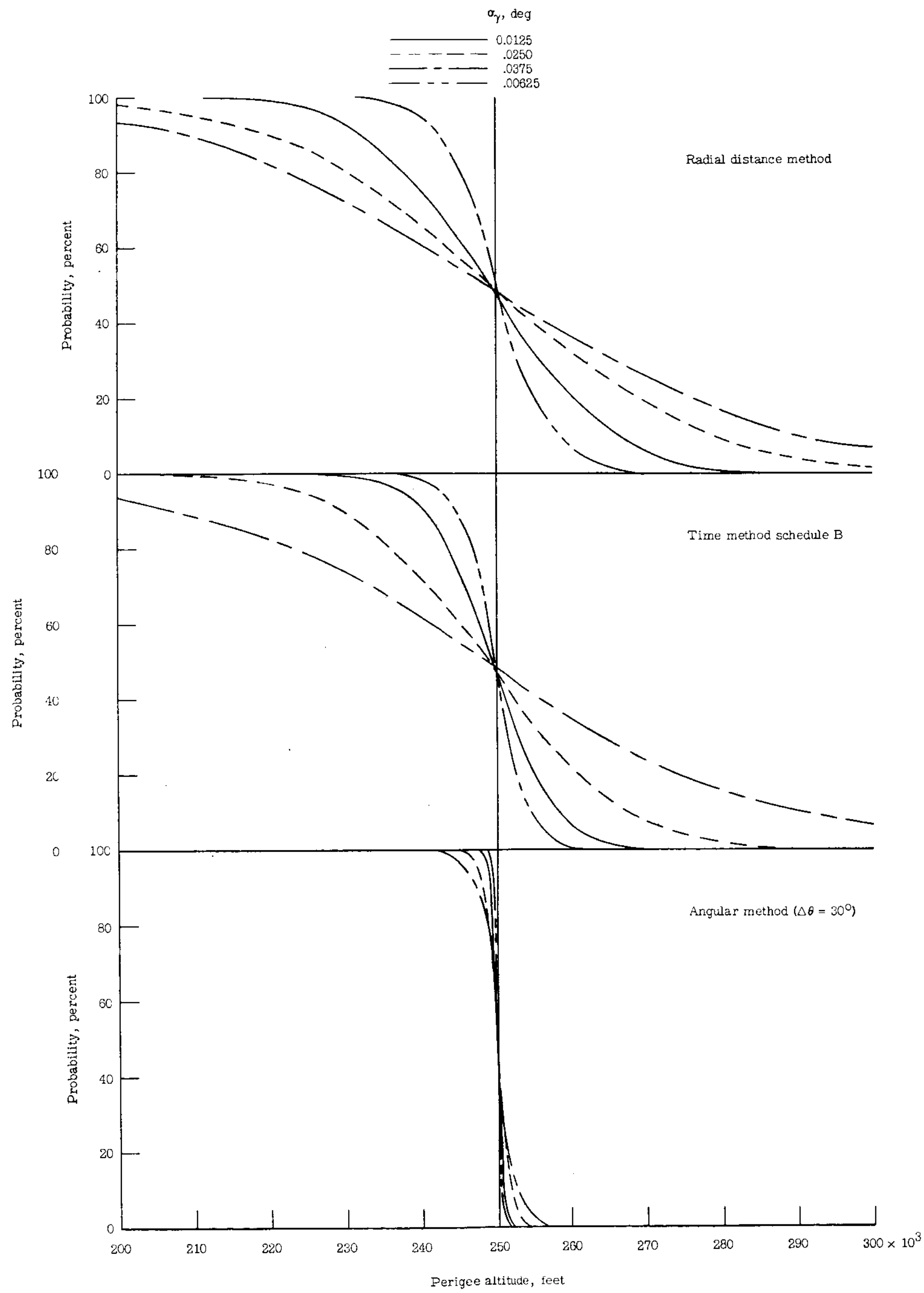

Figure 18.- Probability of exceeding a given perigee altitude for four errors in γ that were not dependent upon range and where the errors in V and V_T were zero. a_o = 100,000 miles; r_{p_o} = R + 250,000 feet.

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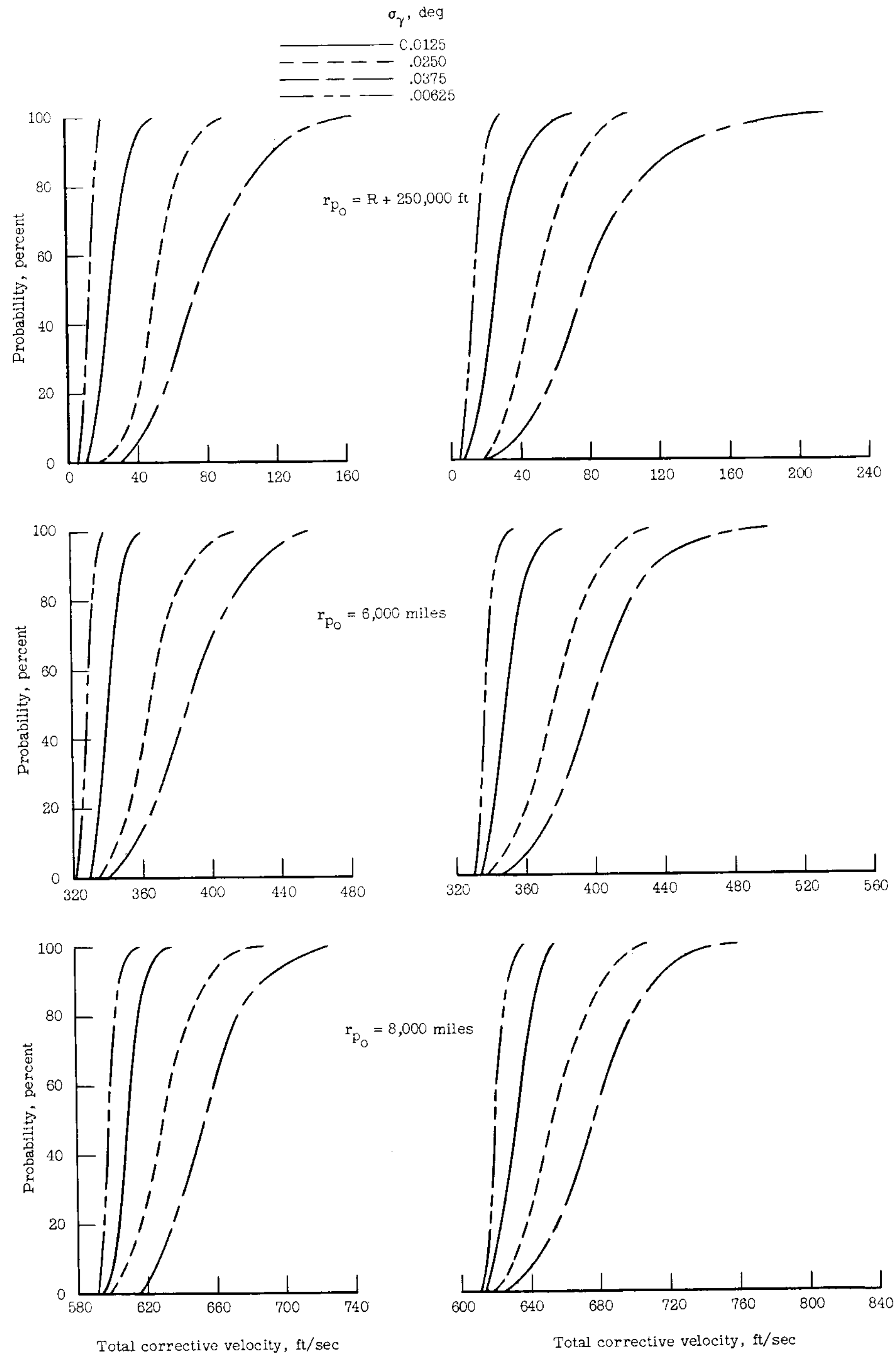

(a) Radial distance method. (b) Time method schedule B.

Figure 19.- Total corrective velocity probability curves for four errors in γ that were not dependent upon range and where the errors in V and V_T were zero. a_o = 100,000 miles.

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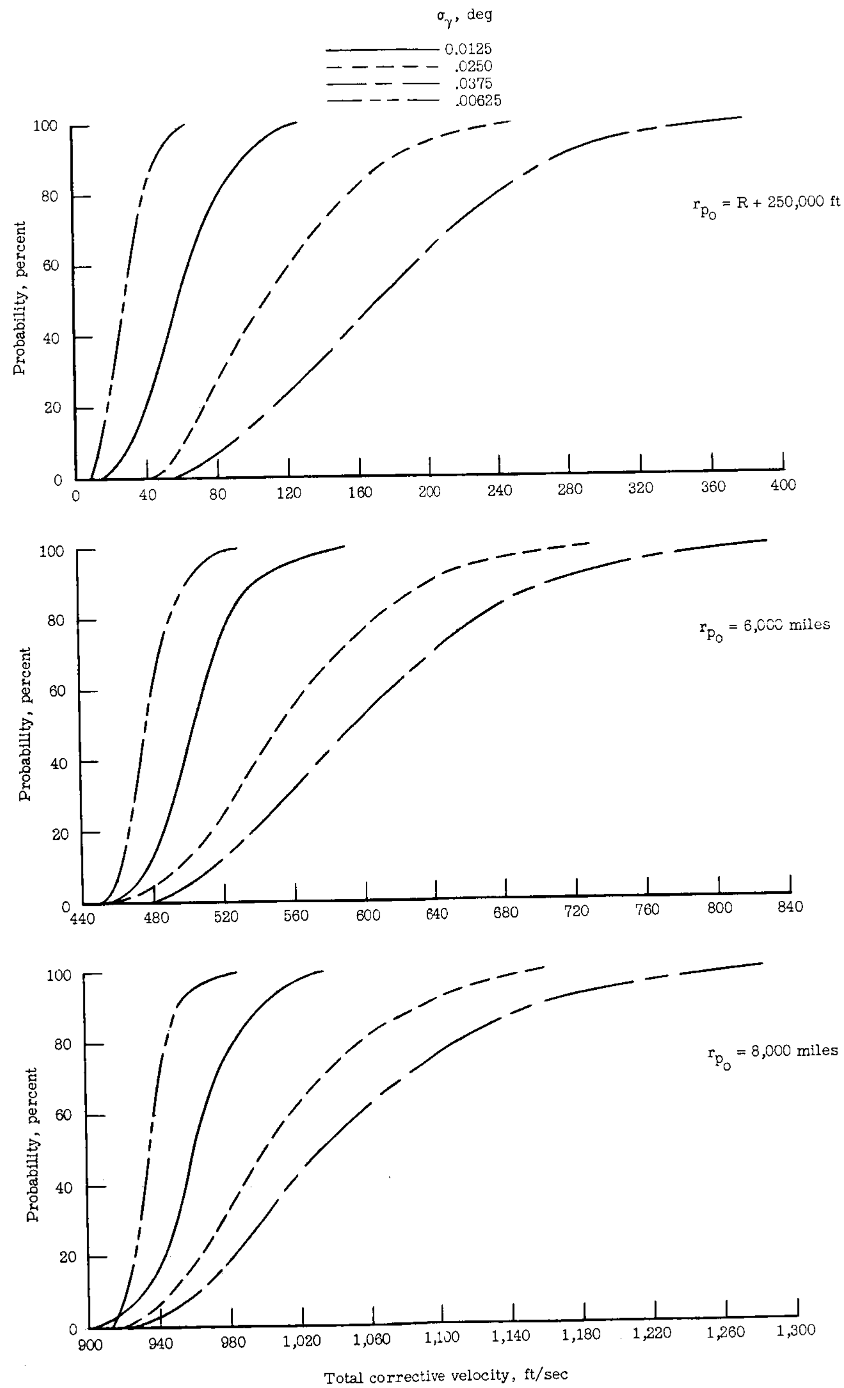

(c) Angular method ($\Delta\theta = 30^{\circ}$).

Figure 19.- Concluded.

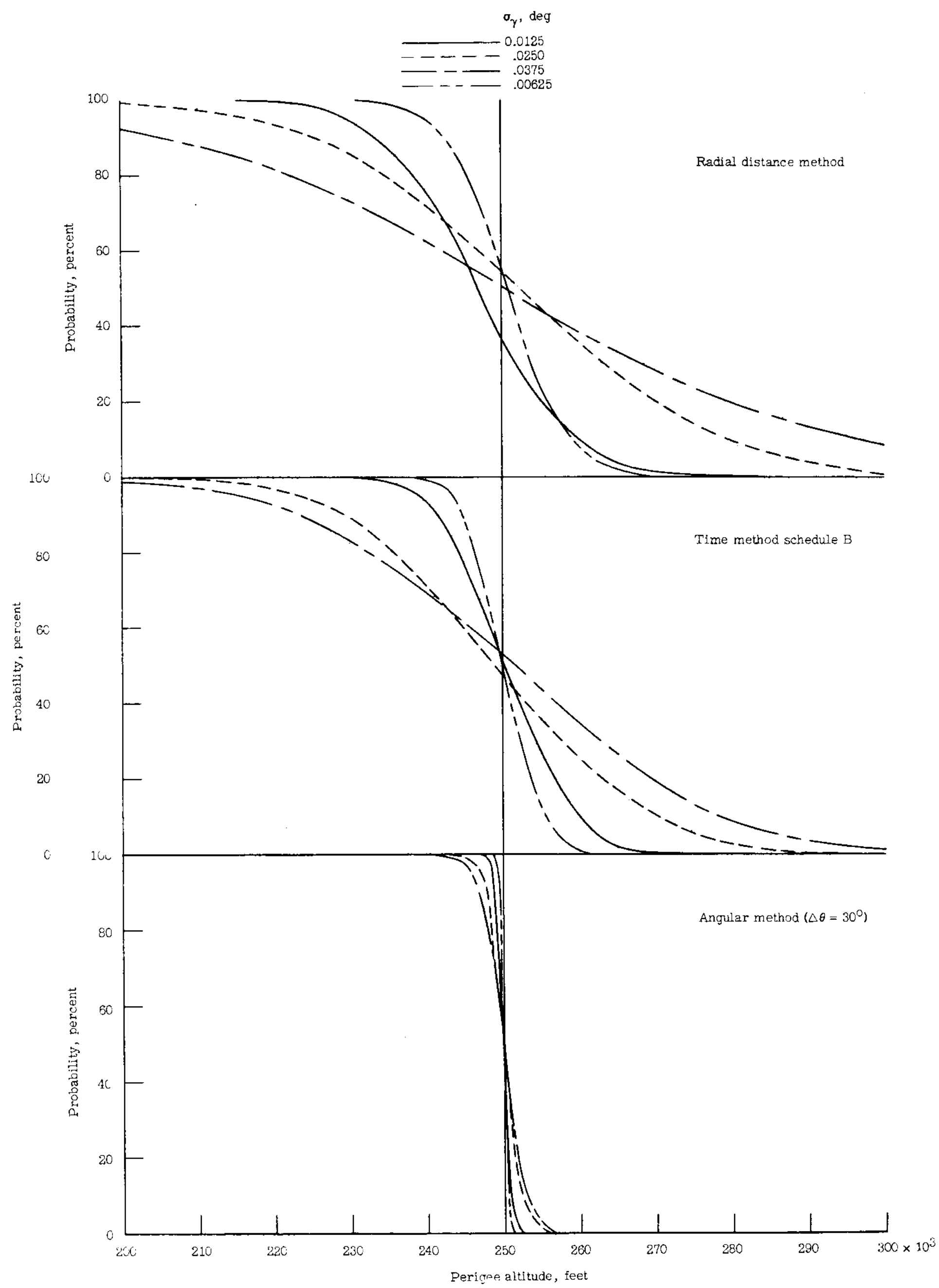

Figure 20.- Probability of exceeding a given perigee altitude for four errors in γ that were not dependent upon range and where the errors in V and V_T were $\sigma_V = 1$ ft/sec and $\sigma_{V_T} = 1.3$ percent.

$a_o = 100{,}000$ miles; $r_{p_o} = R + 250{,}000$ feet.

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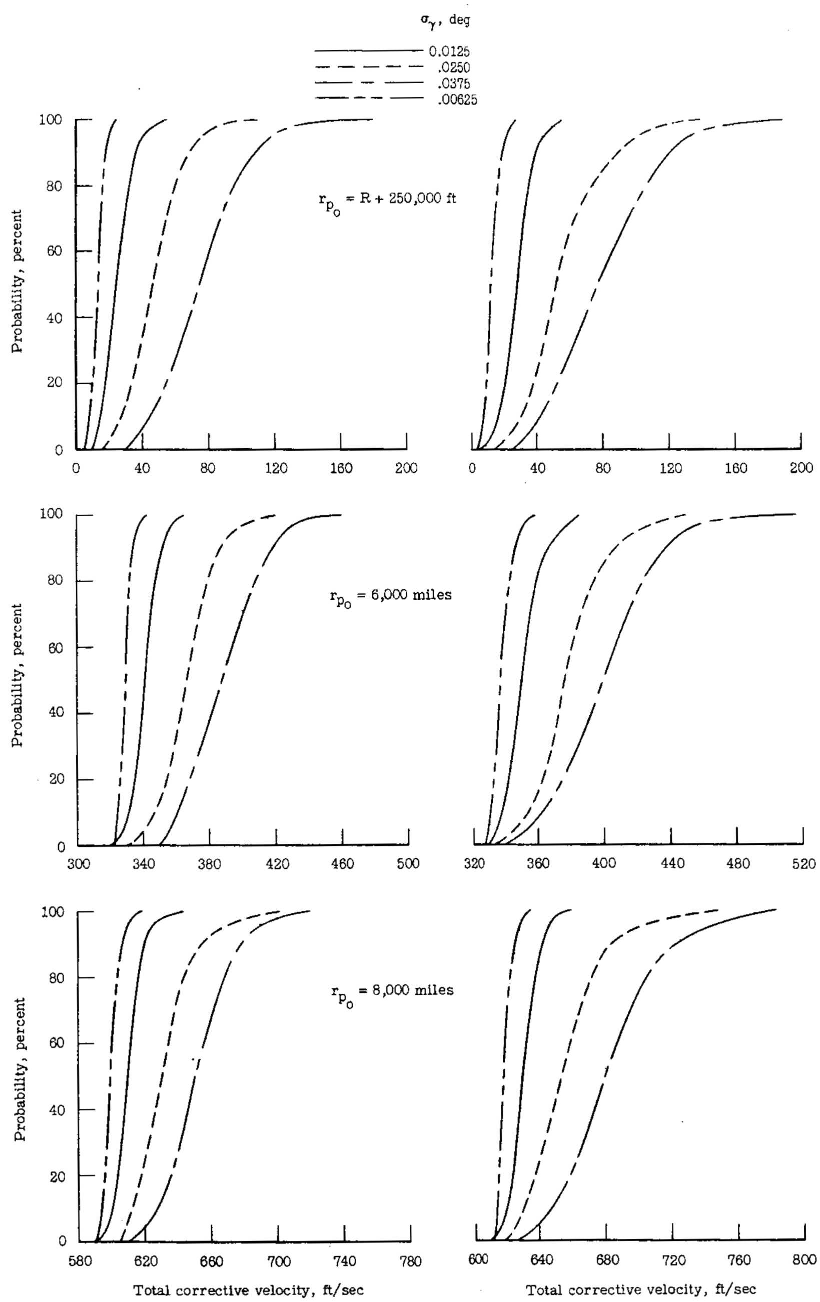
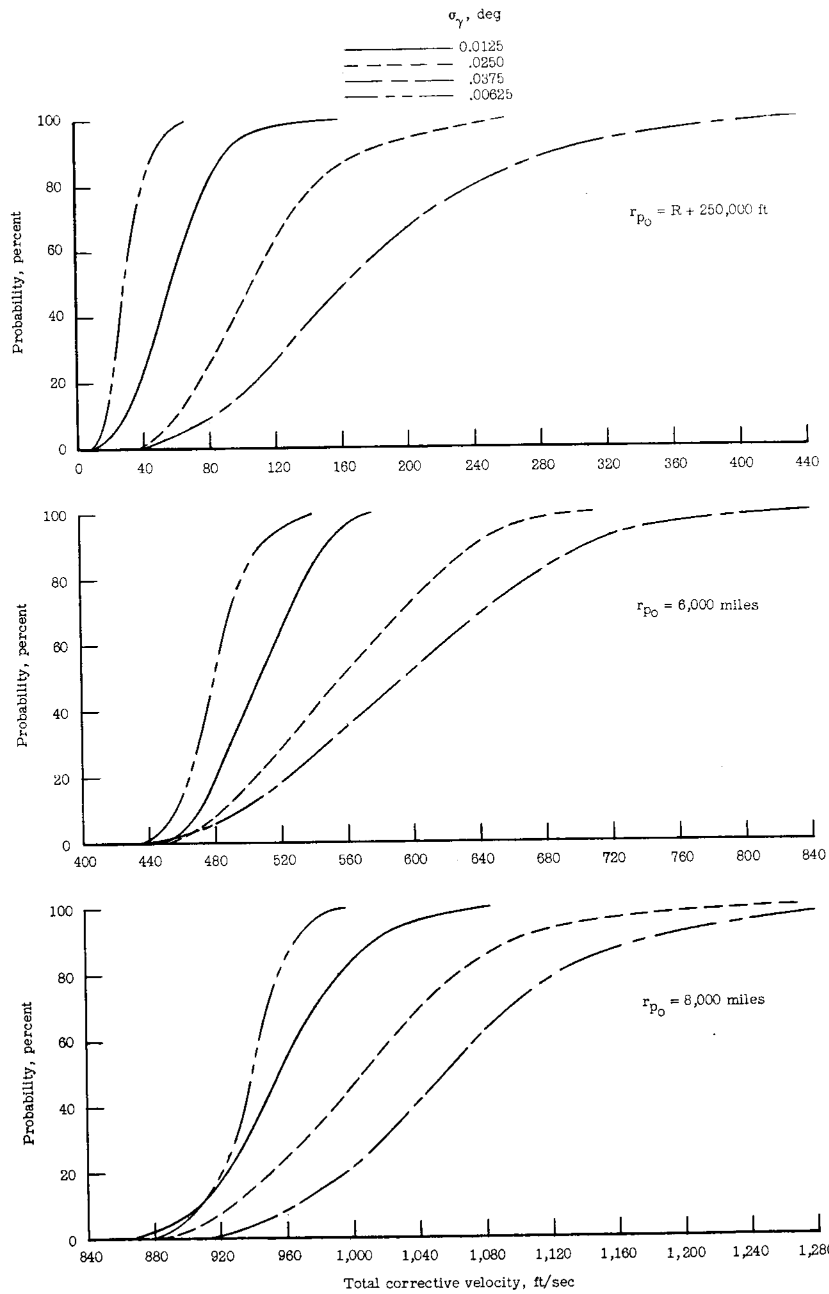


(a) Radial distance method. (b) Time method schedule B.

Figure 21.- Total corrective velocity probability curves for four errors in γ that were not dependent upon range and where the errors in V and V_T were $\sigma_V = 1$ ft/sec and $\sigma_{V_T} = 1.3$ percent.

a_o = 100,000 miles.



(c) Angular method ($\Delta\theta$ = 30°).

Figure 21.- Concluded.

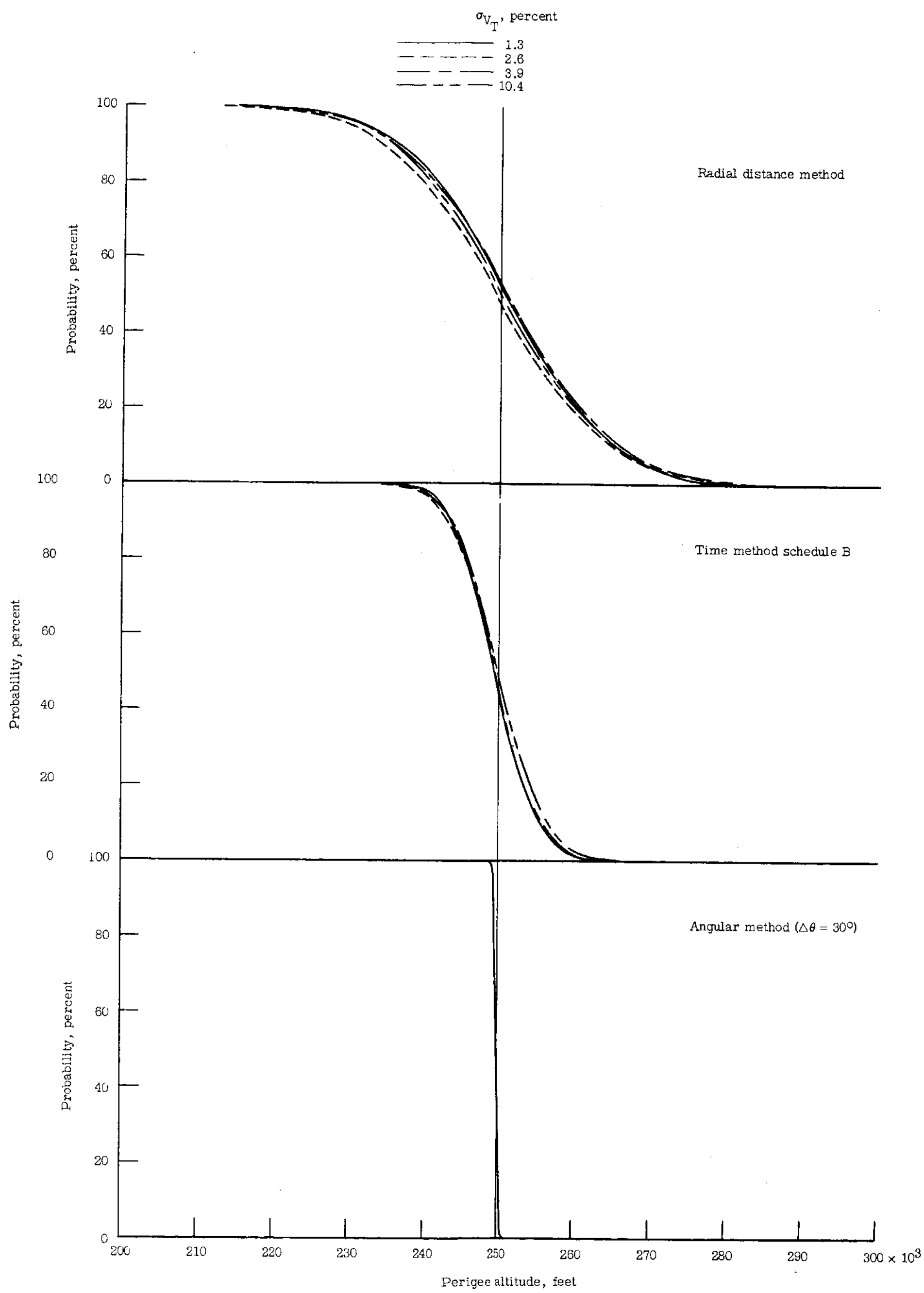
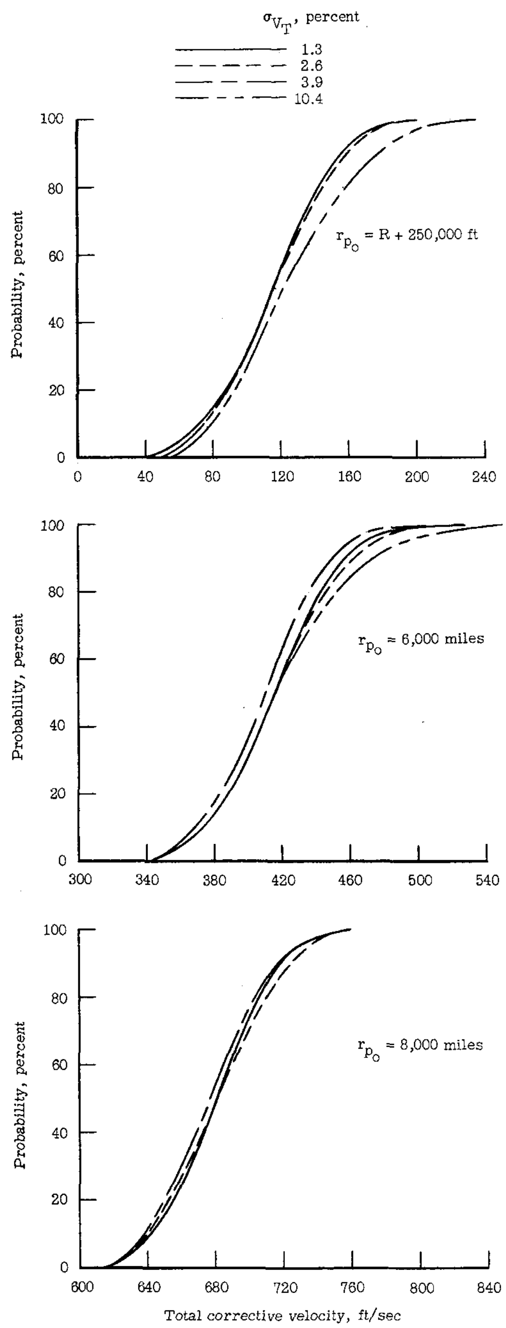


Figure 22.- Probability of exceeding a given perigee altitude for four errors in V_T and where the errors in V and γ were $\sigma_V = 1r/10{,}000$ ft/sec and $\sigma_\gamma = 0.0125r/10{,}000$ degree. $a_o = 100{,}000$ miles; $r_{p_o} = R + 250{,}000$ feet.



(a) Radial distance method.

Figure 23.- Total corrective velocity probability curves for four errors in V_T and where the errors in V and γ were $\sigma_V = 1r/10{,}000$ ft/sec and $\sigma_\gamma = 0.0125r/10{,}000$ degree. $a_o = 100{,}000$ miles.

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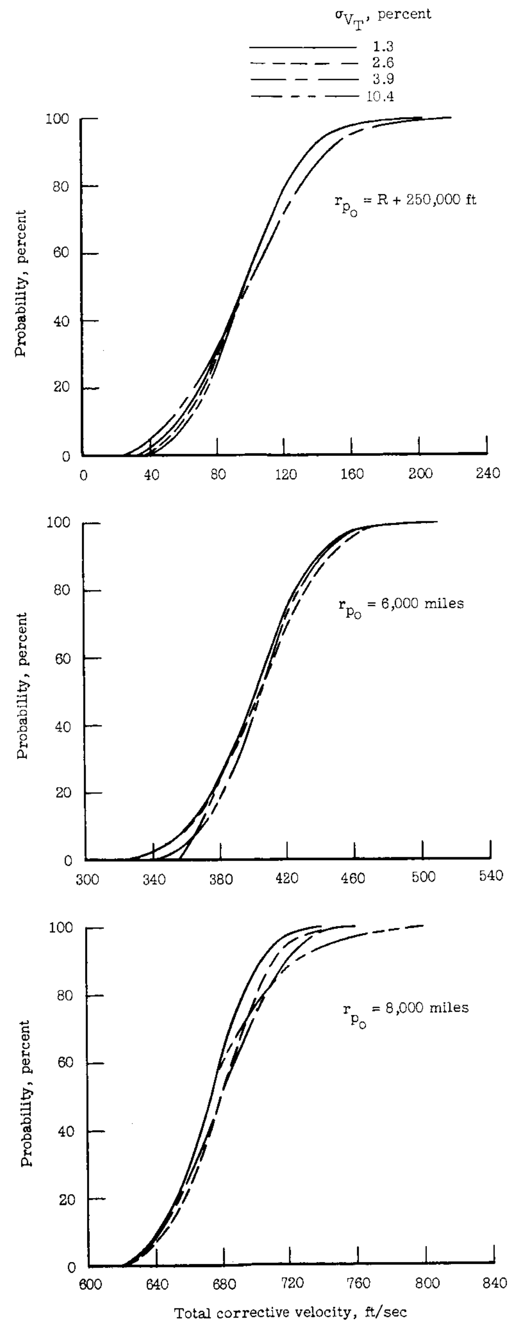
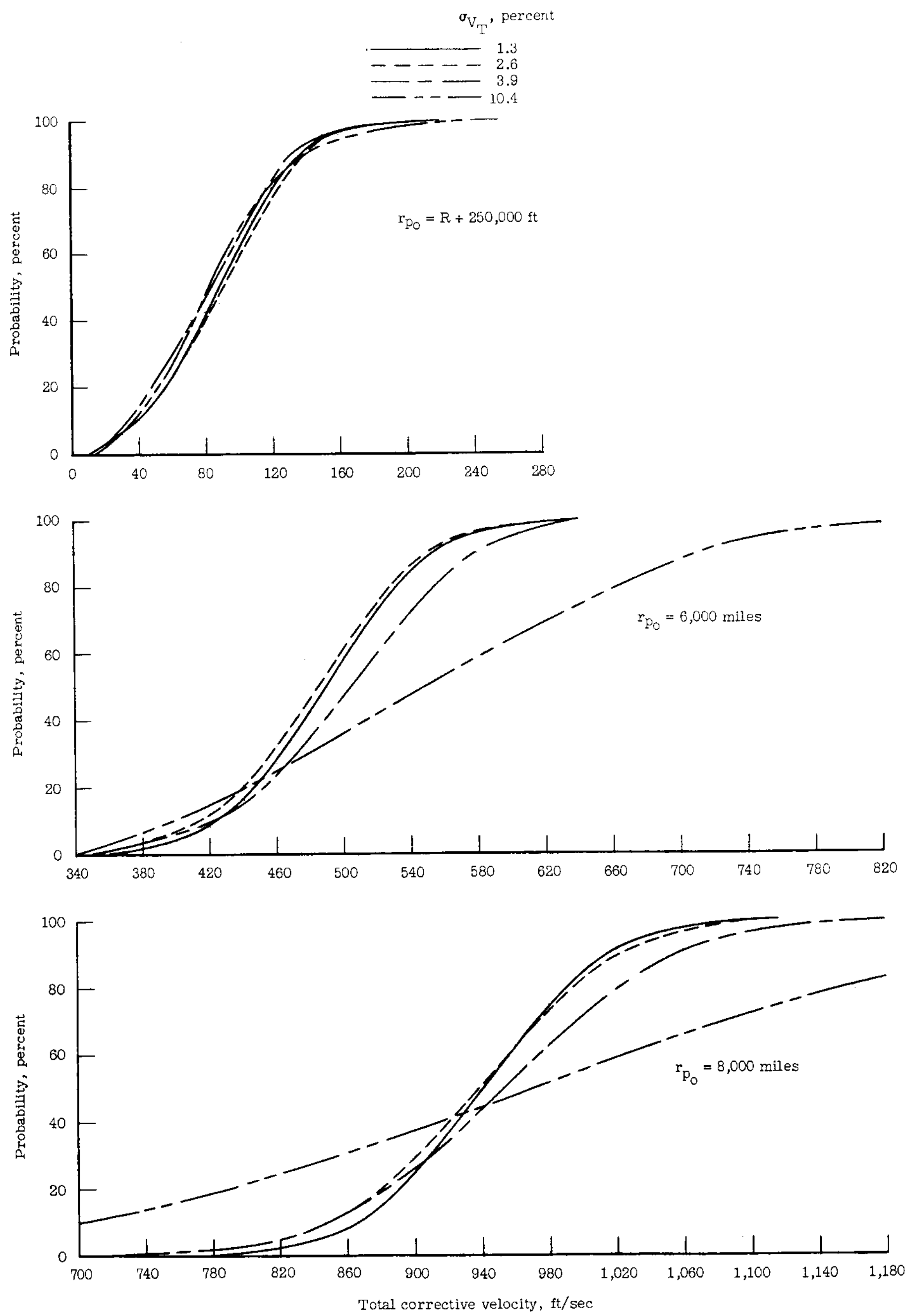


(b) Time method schedule B.

Figure 23.- Continued.



(c) Angular method ($\Delta\theta = 30^{\circ}$).

Figure 23.- Concluded.

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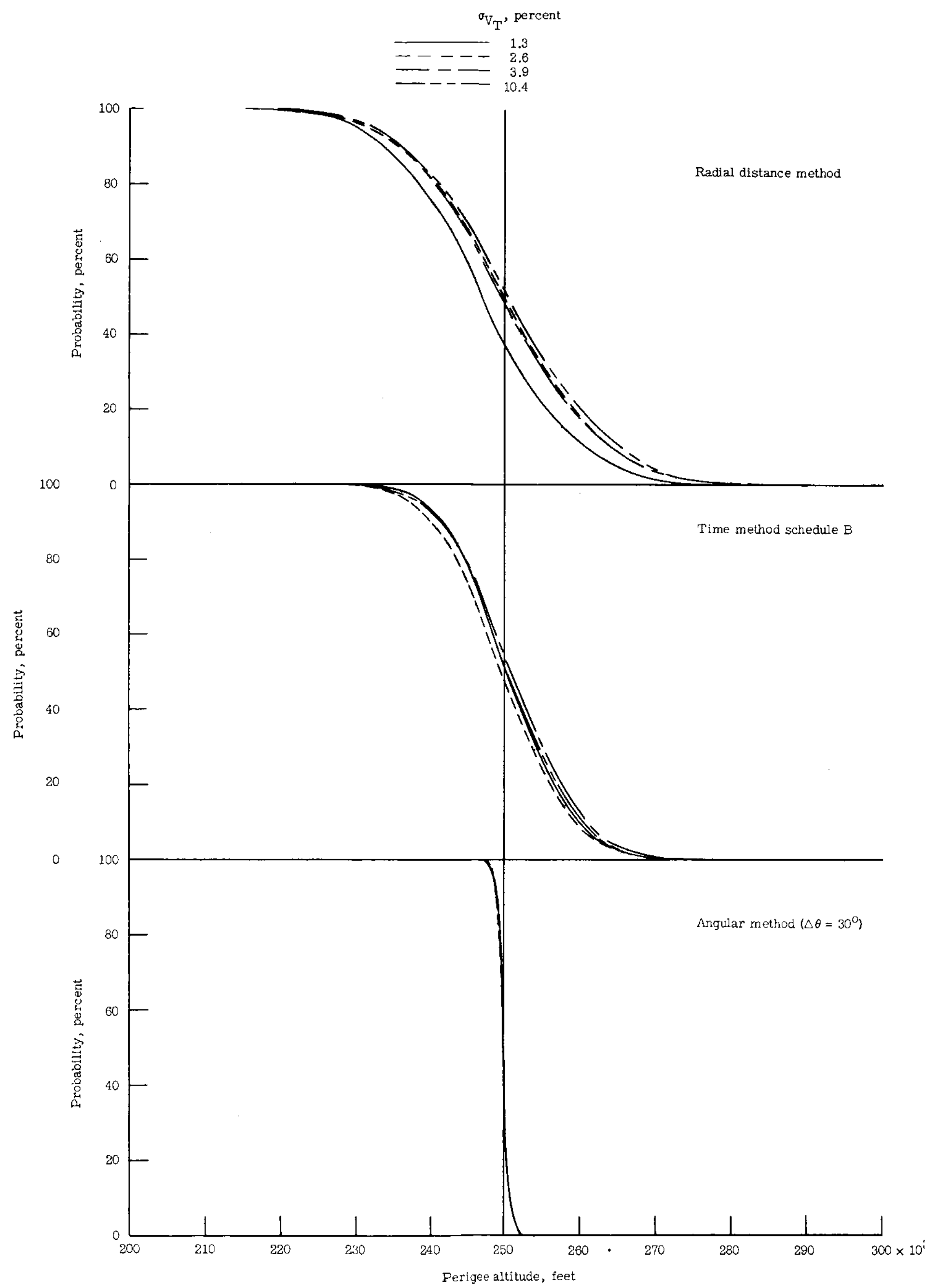
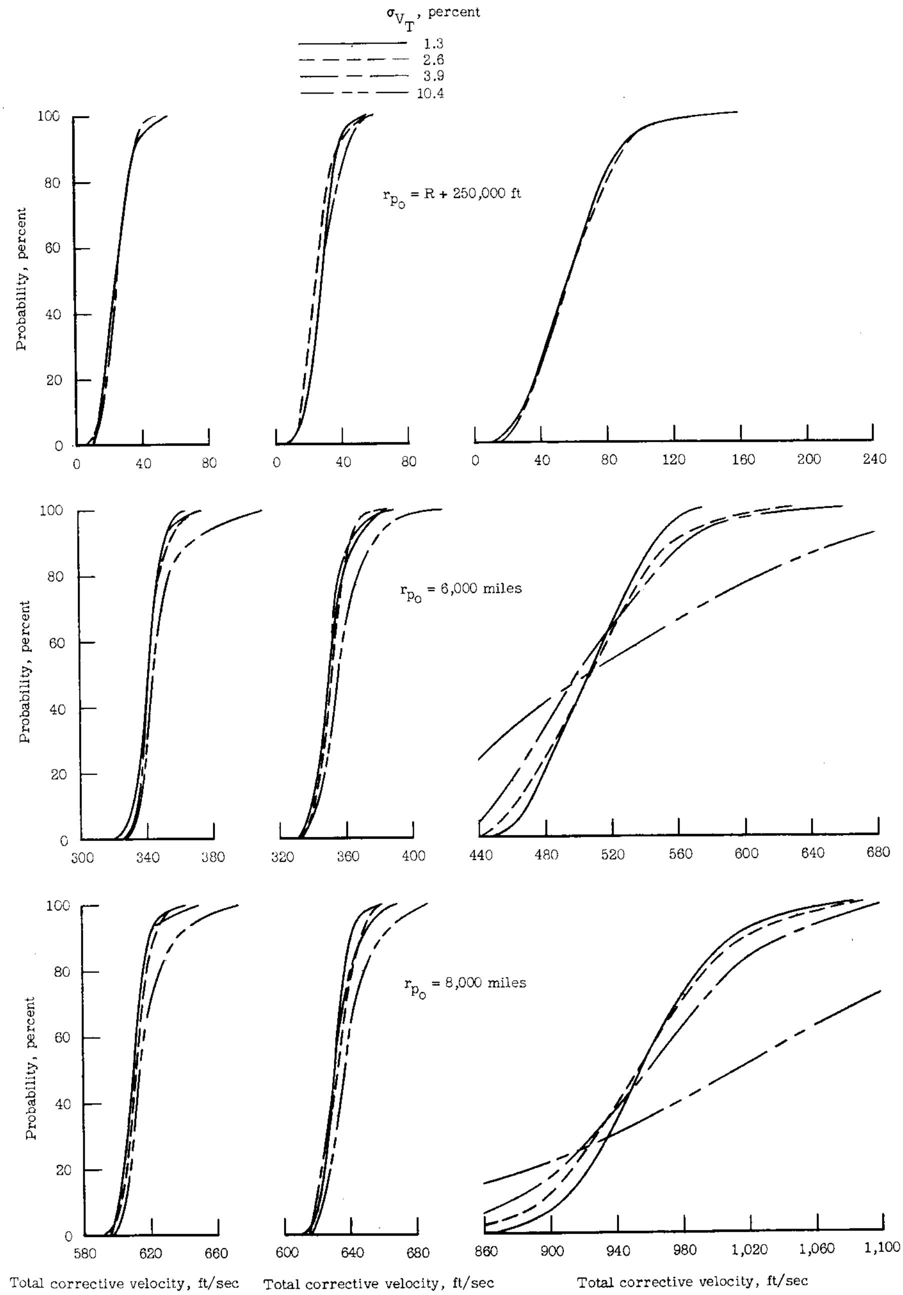


Figure 24.- Probability of exceeding a given perigee altitude for four errors in V_T and where the errors in V and γ were $\sigma_V = 1$ ft/sec and $\sigma_\gamma = 0.0125^\circ$. $a_o = 100{,}000$ miles; $r_{p_o} = R + 250{,}000$ feet.



(a) Radial distance method.

(b) Time method schedule B.

(c) Angular method ($\Delta\theta$ = 30°).

Figure 25.- Total corrective velocity probability curves for four errors in V_T and where the errors in V and γ were σ_V = 1 ft/sec and σ_γ = 0.0125°. a_o = 100,000 miles.

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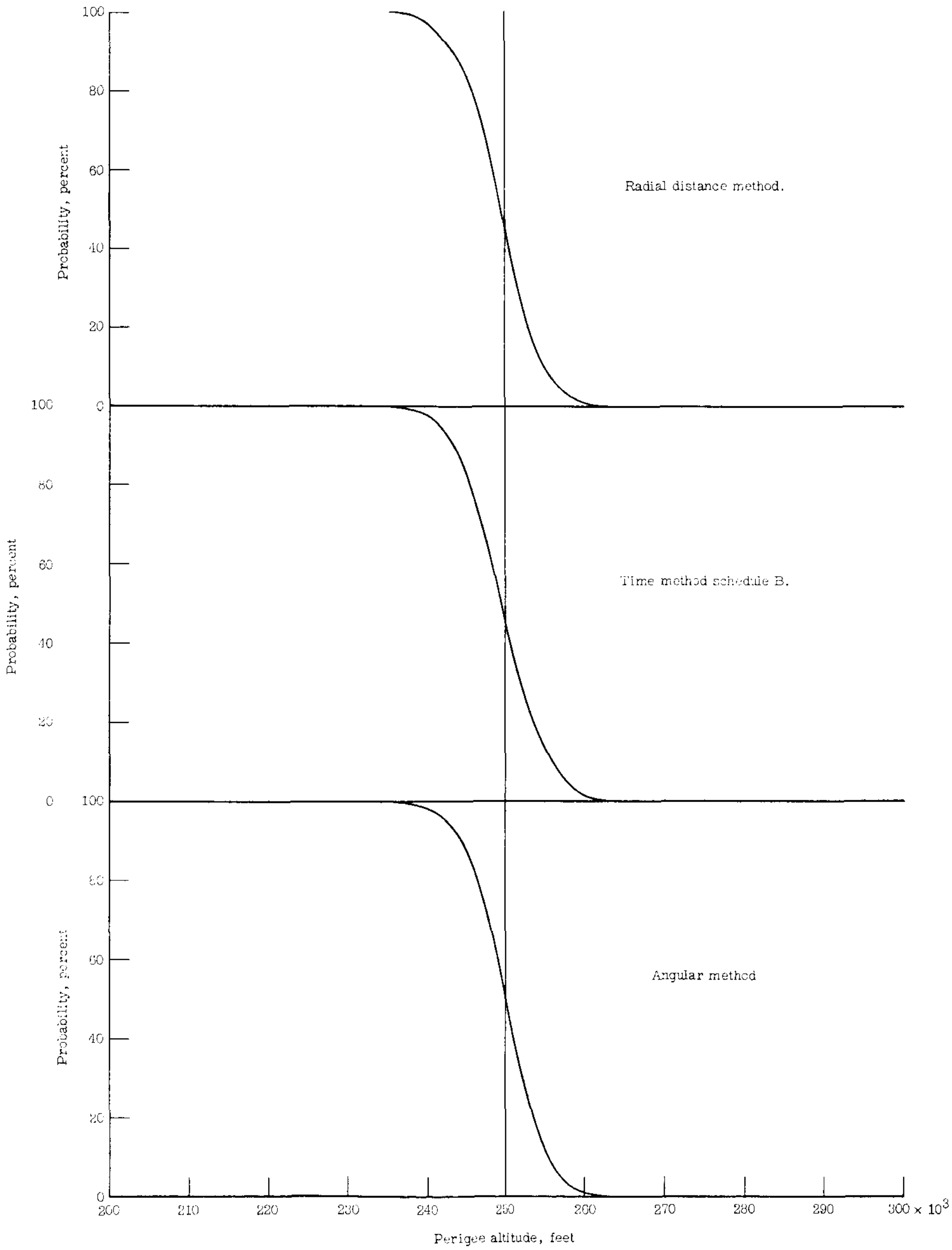

Figure 26.- Probability of exceeding a given perigee altitude for the three methods of scheduling corrective thrust where the radial distance to the final correction point is approximately 6,725 miles. $\sigma_V = 1r/10{,}000$ ft/sec; $\sigma_\gamma = 0.0125r/10{,}000$ degree; $\sigma_{V_T} = 1.3$ percent; $a_o = 100{,}000$ miles; $r_{p_o} = R + 250{,}000$ feet.

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